

Integrating Mathematics and Music in Education: After 60 Years, Can We Reunite the Separated Twins?

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In the past eight years, I've conducted a series of studies on music themed mathematics education together with my colleagues in Texas A&M University and The University of Texas at El Paso, which has resulted in over ten publications in peer-reviewed journals. One of the two questions that I have consistently asked of almost all the participants throughout these research studies has been: "What is your view about the relationship between music and mathematics?" The other consistently present question, phrased in numerous ways, is: "How can we integrate teaching mathematics with music?" If I, myself, had ever been asked these same questions before beginning this line of inquiry focused on the research topic of music-mathematics integrated education, my answers would probably have been very similar to many of classroom teachers who have responded to our queries with trivially superficial answers, such as: "I can use the song *five monkeys jumping on the bed* to teach counting" or "I can use half notes and quarter notes to teach fractions."

While studying at a musical conservatory as a college student, with a specialization in instrument design technologies and acoustical engineering, I first learned of the potential power to create music by understanding mathematics. After repeatedly taking apart a piano featuring over 8,000 individual parts, and then putting them back together like a giant three-dimensional puzzle, I finally managed to tune the more than two-hundred-twenty unique strings by equal tempering an octave of chromatic scale, and then finding a perfect unison and octave for all 88 keys. During such experiences, I came to recognize that the relationship between mathematics and music is more than just one of being interrelated. Instead, music and mathematics is like a pair of symbiotic organisms, and the ability to understand one through the lens of the other has helped contribute towards many of the great musical and mathematical breakthroughs that have occurred during the two field's respective development.

Historically, musicians and mathematicians have collaborated together towards achieving mutual objectives, such as refining acoustics, standardizing auditory evaluation equipment, and creating musical instruments employing complex mechanical and electronic systems. As an example, some of the earliest successful collaborations between musicians and mathematicians can be traced back to around 6,000 B.C. when one of the first manmade musical instruments was constructed utilizing animal bones to create flutes with

complete harmonic intervals. These musical instrument relics were excavated in the 1980s from an early Neolithic tomb located in Henan, China. Because of the many overlaps existing across music and mathematics, numerous professional organizations, research conferences, and academic journals have been developed with varying degrees of emphasis upon examining this intersection. These attempts at establishing a more foundational connection by mathematics and music education have included efforts by the Society for Mathematics and Computation in Music, the International Congress on Music and Mathematics, and the *Journal of Mathematics and Music*. Within these contexts, a self-organized group of musicians and mathematicians have investigated and discussed a variety of topics linking the two subjects, such as: (a) algorithmic and computational approaches within musical scales (Krantza & Douthett, 2011); (b) algebra within periodic rhythms and scales (Amiot & Sethares, 2012); (c) topology of musical data (Sethares & Budney; 2014), (d) logarithms and differential equations in equal temperament (Cho, 2003), and (e) mathematical patterns of certain classical music (Conklin, 2010).

However, most of these relationships connecting music and mathematics have been explored primarily from a basic research framework, with only a very limited subset of the investigations having focused on applied research such as developing music-mathematics connections as a pedagogical resource to facilitate teaching engaging K-12 mathematics lessons that are contextualized within meaningful music-themed activities. This has led to the conclusion that although the natural connections between music and mathematics are rich, only a limited number of classroom teachers have the resources necessary to be able to go beyond music-math education wherein the music-theme is more than just a cover-story adding entertainment value, but rather is a fundamental aspect of the mathematics instruction and accompanying teaching strategies.

Fundamentally, music and mathematics are twin fields—different disciplines for examining similar topics. They are both chiefly interested in examining pattern recognition, with the primary difference that musical pattern recognition generally occurs within the medium of time, and mathematical pattern recognition generally occurs within the medium of timelessness. However, despite their many similarities, these twin fields have been separated within the K-12 school curriculum since the post-Sputnik era began in 1957. With the remarkable emphasis now placed by government agencies and educators upon STEM education, school curriculum has been plunged into a rigid, discipline-based approach. The arts, especially music, have been intentionally marginalized and oftentimes utterly ignored by both teachers and educational policy makers. Yet strong empirical evidence consistently indicates that current mathematics education methods and strategies are not serving students well, evidenced by the small percentage of high school graduates that are properly prepared to enter and succeed within an undergraduate STEM

degree. Of the obstacles these students face, fluency with mathematics appears to be the primary barrier (Sadler & Tai, 2007).

By fortifying the disciplinary boundaries that separate pure mathematics from its application in such fields as music, mathematics educators are blocking the transfer of knowledge that would allow students to experience the real-world power that comes from having mathematics skills. Therefore, mathematics educators should seek to actively discover and extol the connections mathematics and other school subjects, as well as non-school subjects, in order to develop effective mathematics curriculum. Music has shown great potential to be utilized as such a context for developing innovative mathematics teaching strategies that help students build connections between their empirical experiences and the abstract mathematical concepts behind such patterns. In this special issue, twenty-one colleagues plus myself present ten original articles describing the results from our collective line of inquiry into this research domain. These papers provide a body of empirical evidence further continuing the examination of the possibilities and benefits from integrating music into K-16 mathematics education. Perhaps through the continuation of such efforts, one day music and mathematics will be taught together in the school classroom, as the separated twins are finally reunited.

References

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