

Probing Pre-service Teachers' Mathematics Pedagogical Content Knowledge: A Lesson from the Case of the Monotonicity of Function

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Mathematical Pedagogical Content Knowledge (MPCK)'s essence was to transform the academic form of mathematics knowledge into the educational form of mathematics knowledge and its aim was to promote students' mathematics understanding, ability and quality. MPCK could be divided into Topic MPCK and Teaching MPCK. The form of Topic MPCK was to present mathematics questions, and the form of Teaching MPCK was to organize, present and adjust mathematics teaching. In this paper, we studied a lesson about the monotonicity of function that was taught by a Chinese high school pre-service teacher L, and analyzed her Teaching MPCK. Based on the study results, some strategies about improving pre-service teachers' MPCK were given.

Key words: *PCK, MPCK, the monotonicity of function*

Introduction

Teacher knowledge has been a hot topic in the field of teacher education. Many researchers have offered what kinds of knowledge teachers should possess since the 1980s. Among all these study results, PCK is an important knowledge that has had important significance for teacher professional development.

PCK was first set forth by Shulman. Shulman (1987) proposed a framework for analyzing teachers' knowledge that distinguished different categories of knowledge: knowledge of content, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge (PCK),

knowledge of students, knowledge of educational contexts and knowledge of educational ends, purposes and values. Shulman emphasized PCK as a key aspect to address in the study of teaching and defined pedagogical content knowledge (PCK) as the blending of content and pedagogy into an understanding of how particular topics, problems, or issues were organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction. Marks (1990) expanded this definition, making explicit the difference between “an adaptation of subject matter knowledge for pedagogical purposes” and what he termed content-specific pedagogical knowledge, or “the application of general pedagogical principles to particular subject matter contexts”. Graeber (1999) stressed that teachers needed knowledge of why confusions and misconceptions might occur. Grossman (1990) described PCK as the knowledge used to transform subject matter content into forms more comprehensible to students.

Many aspects of knowledge included within PCK have been identified. Shulman (1986) emphasized knowledge of multiple ways of representing the contents to students. Such knowledge relied on the teacher’s understanding of the content, and had as its purpose the transformation of that content into a form that students would understand. Within this idea, Shulman included “illustrations, examples, explanations and demonstrations”. He also included “an understanding of what makes ... topics easy or difficult” as parts of PCK. Van der Valk and Broekman (1999) identified five aspects of PCK: pupil’s prior knowledge, pupil problems, relevant representations, strategies and student activities. An, Kulm, and Wu (2004) emphasized four aspects of PCK: (1) building on students’ math ideas; (2) addressing students’ misconceptions; (3) engaging students in math learning; and (4) promoting students’ thinking about mathematics.

In addition, in this era of globalization and information, new aspects of knowledge, such as knowledge of technology, must be mastered (Angeil & Valanides, 2009).

Based on these above studies, we decided that Mathematical Pedagogical Content Knowledge (MPCK) consisted of Mathematics Knowledge (MK), Pedagogical Knowledge (PK), Content Knowledge (CK) and Technology Knowledge (TK).

Mathematics knowledge - mathematics views, mathematics concepts and proposition, mathematics thought and knowledge of mathematics history.

Pedagogical knowledge - educational views, educational theory (knowledge of education essence, educational aim and so on), knowledge of

curriculum and knowledge of teaching.

Content knowledge - knowledge of students development, knowledge of students' learning attitude, knowledge of students' learning motive, knowledge of students' thinking, knowledge of students' learning strategy, knowledge of students' learning methodology, knowledge of students' learning circumstance.

Technology knowledge—knowledge of traditional teaching media and knowledge of modern education technology (computer).

Furthermore, MPCK's essence was to transform the academic form of mathematics knowledge into the educational form of mathematics knowledge and its aim was to promote students' mathematics understanding, ability and quality. MPCK could be divided into Topic MPCK and Teaching MPCK. The form of Topic MPCK was to present mathematics questions, and the form of Teaching MPCK was to organize, present and adjust mathematics teaching. Teaching MPCK was encased in the teaching plan and instruction. Attentively, MPCK included not only declarative knowledge but also procedural knowledge, namely, MPCK included not only "what" and "how", but also "why".

In light of the above background, this paper reported on an analysis of the teaching MPCK of a pre-service teacher. It would seek to answer the following questions: 1) What MPCK do pre-service mathematics teachers have? 2) What strategies should be used to improve the MPCK of pre-service mathematics teachers?

Method

Subject

The subject was a pre-service mathematics teacher, teacher L. Criteria for inclusion of teacher L in the study were: (1) having four-year education degrees at a famous Normal school; (2) teaching in school districts that had characteristics typical of public schools with respect to the students; (3) having at least half of their teaching experience in high school; (4) willing to provide the data relevant to the reliability and validity of this study, including classroom observations and interviews.

Procedures

In this study, we collected data from multiple sources including classroom observations, semi-structured interviews, lesson plans, teachers' written reflections, students' work samples, and researcher's field notes and so on. We observed two subject matter units for Teacher L using a non-participant observation method. For each unit, at least four class periods were observed. Meanwhile, interviews provided us the context of the teacher's actions and what she knew. In order to save space, in next section, we would research one mathematics lesson about the monotonicity of function that was taught by teacher L.

Backgrounds of the Learning of the Monotonicity of Function

The monotonicity of function was not only an important mathematical concept but also an important character of function in mathematics. There were three cognition phases in which students knew the monotonicity of function. The first, in primary school, students knew some examples. For example, one people's height was increasing along with all increase in his age. The second, in middle school, students knew how to describe how one variable changed along with the other variable. For example, y was increasing along with x 's increasing. The third, in high school, students could abstract examples and generalize the monotonicity of function with mathematics symbol and language. This knowledge and ability was what we would research in this study.

Analysis Framework

Mathematics teaching and learning is a complex process. As we all know, the format of Teaching MPCK was to organize, present and adjust mathematics teaching. Furthermore, the essential components of organizing, presenting and adjusting are shown in table 1:

Table1
The Teaching MPCK

Teaching MPCK	Essential components
Organizing	selecting teaching materials; preparing teaching structure; designing teaching questions; arranging teaching time; managing teaching circumstances.....
Presenting	explanation; example; comparison; symbol; table; figure; graph; model; situation; illustration; manipulation; demonstration.....
adjusting	observing and diagnosing students' learning situation; providing teaching feedback; showing teaching wittiness.....

Since pre-service mathematics teachers were in the early phase of teacher development, there were three basic aspects that they should have mastered: arranging the teaching time rationally; helping students overcome learning difficulties; teaching the problem solving to promote students' thinking.

(1) Teaching time. In China, the teaching time in one lesson is 40 minutes or 45 minutes. Teachers must think how to arrange the teaching time, such as the time for preparing prerequisite teaching situations, the time for learning mathematics concepts, and the time for solving mathematics questions. All these arrangements have a direct influence on teaching efficiency. The level of a teachers' MPCK would be tested regarding these aspects.

(2) Learning difficulties. The learning difficulties were the sections in which students displayed confusion or misconception. So, it is important for teachers to be aware of these learning difficulties. Teachers' MPCK would be tested regarding their way of helping students overcome learning difficulties.

(3) Problem solving. Problem played an important role in mathematics teaching and learning. Appropriate problems could help students understand mathematics. In general, there were multiple ways of solving mathematics problems. So, a teachers' MPCK could be assessed by the process of solving mathematics problems.

In the next section, we analyze teacher L's teaching actions according to the above three views. Based on these teaching actions, we would probe her

MPCK, namely, how did she organize, present and adjust mathematics teaching? How did she transform the academic form of mathematics knowledge into the educational form of mathematics knowledge? How did she promote students' mathematics understanding, ability and quality?

Results

Teaching Episodes and Analysis

The Teaching Time

In this lesson, teacher L began the teaching with two function graphs, $y = x^2$, $y = x^3$, then, she elicited the definition of monotone increasing function and monotone decreasing function through the groups of $y = x^2$, $y = x^3$. After this instruction, she used two methods to judge the monotonicity of function and gave one exercise. Finally, she summarized the lesson briefly.

Teaching Example 1

Table 2

The Teaching Phases and Corresponding Teaching Time

	The Teaching Phases	Teaching time
reviewing	Drawing the graphs of $y = x^2$, $y = x^3$	4'23"
	(i)through the groups of $y = x^2$, $y = x^3$, eliciting the definition of monotone increasing function and monotone decreasing function	3'12"
explaining concept	(ii)writing the definition of monotone increasing function and monotone decreasing function on the blackboard	2'50"
	(iv)analyzing the definition of monotone increasing function and monotone decreasing function	3'36"
	(iv)giving the graph trend about the monotone increasing function and the	2'10"

	monotone decreasing function	
	(v) giving the definition of the monotonicity of function and its monotone interval	1'46"
judging the monotonicity of function	(i) the first method: observing the function graph	6'00"
	(ii) the second method: using the definition of the monotonicity of function	8'06"
summarizing	summarizing the learning content in this lesson	1'28"
exercising	letting students do exercise 1 in the textbook, page 59	3'29"

※ The total teaching time was 40 minutes. There were 2 minutes left in this lesson and teacher L asked students to learn by themselves.

Analysis 1

Because the total teaching time was only 40 minutes, teachers should arrange the limited teaching time rationally. As to the above teaching time, we thought there were at least three aspects to be discussed:

First, teacher L drew the graphs of $y = x^2, y = x^3$ on the spot and this teaching action took up more than 4 minutes. On the one hand, if students were all familiar with the function $y = x^2$. So, why didn't she show the graph directly through the computer, or draw the graph on the blackboard before class? The above teaching action reflected that teacher L should improve the MPCK, the manner of presenting, such as demonstration. On the other hand, if students weren't familiar with the function $y = x^3$. Then she should spend more time drawing the graph of function $y = x^3$ on the spot. If teacher L graphed some functions, such as the function $y = x, y = \frac{1}{x}$, which students were familiar with, it should save some teaching time. The above teaching action reflected that teacher L should improve the MPCK, the manner of organizing, such as selecting teaching materials.

Second, when teacher L wrote the definition of the monotone increasing function and the monotone decreasing function on the blackboard, the process was tedious and prolix. We thought the teacher could also display the definitions directly through the computer, or write it on the blackboard before class. In fact, there were some teaching actions that the teacher could perform before class. These teaching actions also reflected that teacher L should improve the MPCK, the manner of presenting, such as demonstration.

Third, the students' learning difficulty in this lesson was to describe the

monotonicity of function with mathematics symbol and language. This learning process must rely on some visual materials. That is to say, the process about eliciting the definition of monotone increasing function and monotone decreasing function through the graphs of $y = x^2$, $y = x^3$ was very important. But in the class, this learning process only took 3'12". Obviously, the time was not enough for students to understand the correlative learning difficulty.

In general, in the limited teaching time, the teacher should make the best of every minute. For example, doing some preparatory work before class, omitting some questioning that students had mastered, elaborating some questions according to students' prior knowledge and cognition, leaving some questions to discuss after class. All these needed teachers' abundant MPCK.

The Learning Difficulties

As we know, students' learning difficulty in this lesson was to describe the monotonicity of function with mathematics symbols and language. Let us see some teaching examples:

Teaching Example 2

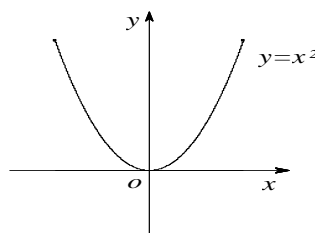
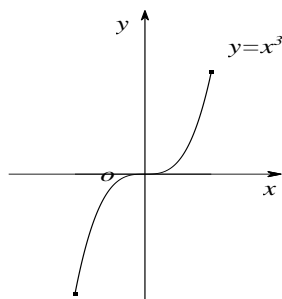


Figure 1. *The graph of $y = x^2$*

Figure 2. *The graph of $y = x^3$*

T (teacher L): These are the graphs for $y = x^2$, $y = x^3$. Next, we will learn the monotonicity of function through the two graphs. First, look at Figure 1, the right of x axis, in the interval $(0, +\infty)$, the trend of the graph is increasing, all right?

S (students): Yes.

T: That is to say, the value of y is increasing along with the x 's increasing. So, if we use mathematics language to describe the phenomenon, namely, take two points in the interval $(0, +\infty)$ randomly, and suppose their horizontal

coordinates are x_1 and x_2 respectively, their vertical coordinate are y_1 and y_2 respectively. We can see $x_1 \dots\dots$

S: $x_1 < x_2$.

T: What is the relationship between y_1 and y_2 ?

S: $y_1 < y_2$.

T: Ok, we say that $y = x^2$ is a monotone increasing function in the interval $(0, +\infty)$. Next, let us see the left of x axis, in the interval $(-\infty, 0)$. What do we notice about is the trend of the graph if we see it from left to right?

S :.....(can't hear them distinctly)

T: The trend of the graph is decreasing, all right?

S: Yes.

T: In other words, along with the x 's increasing, the value of y is?

S: the value of y is decreasing.

T: Yes. If we take two points in the interval $(-\infty, 0)$ randomly, and suppose their horizontal coordinates are x_3 and x_4 respectively, their vertical coordinates are y_3 and y_4 respectively. We can see $x_3 \dots\dots$

S: $x_3 < x_4$.

T: What about y_3 and y_4 ?

S: $y_3 > y_4$.

T: So, we say that $y = x^2$ is a monotone decreasing function in the interval $(-\infty, 0)$. Based on the above analysis, please judge the monotonicity of $y = x^3$ in the interval $(-\infty, +\infty)$.

S: The monotone is increasing.

T: Why? (Not waiting for students to answer this question, teachers L answers this question on her own) First, the trend of the graph is increasing. If we take two points, and $x_1 < x_2$, then $y_1 < y_2$, yes or no?

S: Yes.

T: So, we say $y = x^3$ is a monotone increasing function in the interval $(-\infty, +\infty)$. Now, we have two new definitions: monotone increasing function and monotone decreasing function.

Analysis 2

The way of helping students to overcome the learning difficulties was to evaluate the lesson. In other words, the success of the teaching regarding the monotonicity of function relied on if the students could transform the observation conclusions about those dynamic graphs into static mathematics symbol and language.

If teachers wanted to help students to overcome the learning difficulty in this class, the learning process should be broken up into two steps: first, let students observe the trend of specific function graphs and allow students to form the feature about the monotonicity of function. Second, led students generalize the feature about the monotonicity of function. In addition, in the learning process of the definition of the monotonicity of function, the randomness of x should be emphasized and the fact that there were two meanings: one was to use a static mathematics symbol to describe the dynamic trend of function graphs; the other was to use some single points to describe the whole definition domain. This question was only understood in the learning process when students met learning contradictions. For example, as to $y = x^2$, if one point was taken from the interval $(-\infty, 0)$, and the other point was taken from the interval $(0, +\infty)$, when $x_1 < x_2$, there might be three results: $y_1 < y_2$, $y_1 = y_2$ and $y_1 > y_2$. So, when we discussed the monotonicity of function, the interval in the definition domain must be pointed out clearly.

However, the two learning steps about the monotonicity of function and the two meanings of the randomness of x weren't shown in the learning process. As to the learning difficulty in this class, "how to describe the monotonicity of function with mathematics symbol and language," students still didn't understand it. They only memorized the exterior mathematics symbol mechanically.

Furthermore, in the above teaching process, teacher L controlled the teaching flow entirely; students had few opportunities to express their views. Outwardly, teacher L asked some questions and asked students to answer these questions. In fact, students only needed to "fill a vacancy". There was no space for students to explore. All the above teaching actions reflected how teacher L should improve the MPCK, the manner of adjusting, such as observing and diagnosing students' learning situation.

MPCK's essence was to transform the academic form of mathematics knowledge into the educational form of mathematics knowledge and its aim was to promote students' mathematics understanding, ability and quality. So, teachers must pay attention to students' thinking and build on students' mathematics ideas. Obviously, teacher L's teaching actions couldn't promote students' mathematics thinking further.

(3) The problem solving

After introducing the first method, which was to judge the monotonicity of function by observing the function graph, teacher L didn't give any transition and went to the second method, which was to use the definition to

prove the monotonicity of function. Let us examine some teaching examples:

Teaching Examples 3

T: Next, let us use the definition of the monotonicity of function to prove the question:

Question: Prove the function $f(x) = 3x + 2$ is monotone increasing, $x \in R$.

How do we use the definition of the monotonicity of function to prove the question? First, take x_1, x_2 belonging to one interval in the definition domain randomly, and suppose $x_1 < x_2$. Second, judge $y_1 > y_2$ or $y_1 < y_2$. Thus, the monotonicity of function could be obtained.

For this question, $\forall x_1, x_2 \in R$, and $x_1 < x_2$, $f(x_1) - f(x_2) = (3x_1 + 2) - (3x_2 + 2) = 3(x_1 - x_2)$, so, we could get $f(x_1) - f(x_2) < 0$, that is to say, $f(x_1) < f(x_2)$, and $f(x) = 3x + 2$ is a monotonic increasing function, all right?

S: Yes.

T: The result is proven through the definition of the monotonicity of function. The steps for solving are as follows: first, take x_1, x_2 belonging to one interval in the definition domain randomly, and suppose $x_1 < x_2$. Second, judge $y_1 > y_2$ or $y_1 < y_2$. In general, we perform $y_1 - y_2$, and see the result is bigger than 0 or smaller than 0. Lastly, we would know the monotonicity of function.

Analysis 3

In the above process,

(1) Students didn't know why the teacher used the definition to prove the monotonicity of $f(x) = 3x + 2$. All students were familiar with the graph of $f(x) = 3x + 2$. Its graph was a line and the trend was distinct. Through its graph, students could easily know it was a monotonic increasing function. This method was visual. In other words, this question did not need to use the method of using the definition to prove its monotonicity and student didn't understand the advantage of this method. Students' confusion came from teacher's insufficient MPCK or teaching organizing. Before solving this question, teacher L didn't give any transition. In fact, the graphs of some functions were easy to draw or were known by us. Thus, we could judge the monotonicity of function through observing their graphs. However, the graphs of some function were difficult to draw or weren't known by us. So, the other method was needed, which was to use the definition to prove the monotonicity

of function. If teacher L wanted students to understand the advantage of this method, she should choose a function for which its graph was difficult to draw or for which students didn't know its graph. All the above teaching actions reflected that teacher L should improve the MPCK, the manner of organizing, such as selecting teaching materials.

Furthermore, MPCK's aim was to promote students' mathematics understanding, ability and quality. Thus, teachers must build on students' mathematics ideas. In other words, in the learning process, teachers must help students to connect new knowledge with prior knowledge and form knowledge structure. This is supported by some scholars, (An et al., 2004, p. X), "Linking the new and prior knowledge in context will also help students know why and how to learn the new topic and grasp new knowledge with better understanding". Thus, the definition of the monotonicity of function should be connected with the expression of the function, the graph of the function and so on. In this learning process, students' thinking is better developed.

(2) When judging $y_1 < y_2$ or $y_1 > y_2$, teacher L emphasized the method through working $y_1 - y_2$. This method was emphasized again in the following mathematics problem solving. In fact, the aim of "working $y_1 - y_2$ " was just to judge $y_1 < y_2$ or $y_1 > y_2$. Thus, the teacher shouldn't restrict students' thinking. For example, in this question, there was another method of judging $y_1 < y_2$ or $y_1 > y_2$:

$$\because x_1 < x_2, \therefore 3x_1 < 3x_2, \therefore 3x_1 + 2 < 3x_2 + 2, \text{ thus } y_1 < y_2.$$

The way of mathematics explanation was varied. Teachers should try their best to elicit students' thinking. This reflects if a teacher possesses a high level of MPCK.

Interviews and Analysis

Interviews

After class, Teacher L and the first author of this paper talked about this lesson for more than an hour. Some portions of their conversation follow:

A (Author): What do you think about this lesson?

T (Teacher): I don't know if the students understand the content and my feeling was not good.

A: I feel the students' learning enthusiasm was high at the beginning of this lesson, but the enthusiasm descended gradually along with the teaching

process.

T: I don't know why it is. Students seemed not to cooperate with me in this class although I think the relationship between us is good.

A: How about is your teaching plan?

T: My teaching plan? I consult other teaching plans and copy the good teaching plans.

A: Did you read the teaching reference book?

T: No, the teaching reference book is too abstract to read.

Analysis

From the above conversation, we could see that teacher L felt the teaching effect was not good, but she didn't find the real reason for the teaching failure. In her opinion, the reason rested on students mostly, for students didn't cooperate with her in class. Her teaching plan was not at fault, for she consulted some good teaching plans. However, why didn't students cooperate with her in class? Did she consider students' prior knowledge and their way of thinking when she consulted other teaching plans? In fact, the real reasons for the difficulties were her mathematics education views and her insufficient MPCK. Namely, she didn't put students into the central role when they were learning mathematics; she didn't pay attention to the interaction between teachers and students; she didn't pay attention to students' learning desire, and she didn't let students think about the questions by themselves and so on. Under these mathematics education views, how could her MPCK be sufficient?

In addition, in her other lessons, such as inverse functions, the teaching actions that we mentioned above still occurred in her class. For example, she told students everything and the students had no time and space to think about the mathematics questions; she paid attention to memory excessively and ignored the process of mathematics question solving.

Conclusions

Many pre-service mathematics teachers have the same teaching actions as teacher L. For example, teacher S, a pre-service mathematics teacher, didn't explain some learning tasks clearly, didn't give students sufficient time for exploring mathematics questions, didn't let students think about the mathematics questions by themselves, didn't know how to lead students to analyze mathematics problems nor how to express learning concepts. Faced with students' answers, teacher S couldn't correct their understanding errors,

and she had no choice but to tell the students some of the answers (Li & Yu, 2010).

In summary, the following teaching actions often occurred in the teaching of some pre-service mathematics teachers: (1) they finished their teaching within 20 minutes and didn't know what to do next; (2) they seldom questioned students and even if they posed a question, there was no time for students to think about the question. In contrast, the question was answered by the teachers themselves; (3) they spent little time explaining mathematics concept, or explained mathematics concepts using exterior language excessively and ignored interior mathematics meaning; (4) they didn't lead students to think about how mathematics knowledge was studied and ignored the inner connection between mathematics knowledge; (5) they paid attention to the technique of problem solving excessively and ignored mathematics thinking about these problems.

All the above teaching actions reflected that these pre-service mathematics teachers' MPCK was not sufficient. Thus, teacher educators should help pre-service mathematics teachers to improve their MPCK.

First of all, teacher educators should pay attention to pre-service teachers' mathematics education views. Fine mathematics educational views might not result in fine teaching actions, but awful mathematics educational views are sure to bring about awful teaching actions.

Then, teacher educators should make pre-service teachers realize mathematics teaching and learning is a very complex process; further, teacher educators should teach pre-service teachers more teaching strategies; teacher educators should introduce pre-service teachers to self-reflection (Li & Yu, 2010).

Furthermore, pre-service mathematics teachers could use the learning philosophy "from imitating to innovating" to improve their MPCK. Imitating is a learning process in which people do the similar actions following some examples. There are two categories in the process of imitating: mechanical imitating and meaningful imitating. Here, we emphasized the latter, meaningful imitating. Meaningful imitating means pre-service mathematics teachers created new ideas based on others' jobs, have own thinking and put this thinking into their own teaching practice.

Excellent teachers have many good teaching experiences that are formed in their long teaching practice and pre-service mathematics teachers should have the opportunity to learn through these good teaching experiences. But the learning must be meaningful. Otherwise, even if pre-service mathematics

teachers used the same MPCK to show the same teaching flow, the learning effect will not be good.

For example, a pre-service mathematics teacher, teacher S, listened to one lesson about the application of mean inequality that was taught by an excellent mathematics teacher, teacher Z. In this lesson, teacher Z used sequences of questions and led students to think step by step. Pre-service teacher S felt this teaching strategy was very good. The next day, in his class, which was about the definition of the monotonicity of function, he spent only 10 minutes explaining the mathematics concept. Then, he used sequences of questions (six questions) and tried to lead the students' thinking. After this class, teacher S said he didn't know why the teaching effect was not good. We couldn't deny that teacher S showed good imitating enthusiasm. But within 40 minutes, students learned not only the mathematics concepts but also had to think about so many mathematics questions: how could they understand all this knowledge? In the class about the application of mean inequality, the teaching key was mathematics knowledge application. It meant students needn't need to learn new mathematics knowledge and only applied the knowledge that they had learned into mathematics problem solving. But in the class about the definition of the monotonicity of function, the teaching key was the definition. It meant that students needed sufficient time to learn the new mathematics concepts. Those series of questions should be put in the following learning. Teacher S posed sequences of questions ahead of time and the prospective teaching effect wasn't achieved. The reason was that teacher S imitated teacher Z mechanically. He didn't think the condition and occasion of utilizing the teaching strategies.

In the learning manner of "from imitating to innovating", pre-service mathematics teachers should pay attention to the following points:

(1) Self-confidence

Self-confidence played an important role in teachers' learning and teaching. Pre-service mathematics teachers must believe that they possess the teaching abilities, they can copy teaching problems, and they can answer students' questions. Thus, they can really face teaching challenges and improve their MPCK rapidly.

(2) Independent thinking

Some pre-service mathematics teachers only copied excellent teaching plans and didn't modify them according to their students' actual learning situation. Some pre-service mathematics teachers depended on their tutor excessively and lacked the consciousness of self-study. All these actions went

against their progress. The success of everything relied mostly on interior factors and the exterior factors had only an assistant role, as did the teachers' learning.

(3) Knowledge of students thinking and Technology Knowledge

On the one hand, when some pre-service mathematics teachers were asked if they thought about students' prior knowledge, thinking style, or the learning difficulty in their teaching, they said they didn't think about these aspects, which belonged to the domain of knowledge of students thinking.

On the other hand, some pre-service mathematics teachers usually explained the definition verbally. Although the teacher might enlighten students' learning, there was a lack of active management of the students. There was especially a lack of constructive thinking with them. If teachers used the "Geometry Sketchpad" in a timely manner, students would have a better understanding of the dynamics of mathematics in symbolic mathematical format and representation (Tu, 2008).

MPCK was the blending of different knowledge, including knowledge of students thinking and technology knowledge. Thus, the improvement of MPCK goes along with the improvement of the knowledge of students thinking and technology knowledge.

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