

Research on Mathematics Instruction Experiment Based Problem Posing

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This research tends to make the experimental study on the mathematics teaching model of "Situating Creation and Problem-based Instruction" (SCPBI), namely, the teaching process of "creating situations posing problems solving problems applying mathematics". It is aimed to change the situation where students generally lack problem-based learning experience and problem awareness. The result shows that this teaching model plays a vital role in arousing students' interest in mathematics, improving their ability to pose problems and upgrading their mathematics learning ability as well.

Key words: mathematics situations, problem posing, problem solving, mathematics teaching, experimental class, control class.

Introduction

In mathematics teaching of primary and secondary schools, teachers usually devise some mathematical problems for students to solve, such as mathematical proof, algebraic computation, numerical inspection etc. Most of them are characterized by their clear statements and definite targets. Obviously, they could have helped students to master mathematical knowledge and skills, however, these problems are far from all mathematical activities. In fact, whether it is a science subject or a mathematics activity, mathematics consists of two aspects: "problem posing" and "problem solving". So, when the "problem" is regarded as the heart of mathematics, it seems to be not only the "problem-solving" object, but also the mathematical creativity which can be found.

In response, "*Full-time Compulsory Education Mathematics Curriculum Standard (experimental draft)*" (MEPRC, 2001) and "*Senior High School Mathematics Curriculum Standard (experimental draft)*" (MEPRC, 2003) promulgated by the Ministry of Education one after another, specify the curriculum target that students should learn how to pose mathematics problems, understand them and solve them from the mathematical angle.

But the real mathematics classroom environment shows us the prospect. First, teachers' classroom questions are in high density and low level, which arouses people's

doubts about this “lack of interactive teaching”. Second, the learning method of getting accustomed to learning to “answer” instead of learning to “pose” is hindering students’ development of problem consciousness and innovation ability. Thereupon, when problem posing has become the important component in mathematics curriculum, and when experimenting, observing, exploring and conjecturing have become the same important mathematics thoughts and methods as logic and abstract, people cast their eyes on the common topic: “Why do our students rarely ask questions?”

Over the past twenty years, teaching research into mathematical problem posing has attracted the popular attention of the field of mathematics education at home and abroad (Xia 2005). The main reason for this lies in the reflection of the current situation on teaching practice of problem solving, and the developing requirement of knowledge and economical society on mathematical initiative talent. The research focuses on how to instruct and encourage students to find out and pose problems with the help of their problem awareness. Consequently, researchers discuss the teaching methods and strategies of “problem posing” from different angles, which offers abundant resources, that is, the transition from curriculum ideas to teaching reality.

However, the research on the “problem posing” in China remains in the beginning phase. In fact, it was not until the promulgation of *National Elementary Education Mathematics Curriculum Standard* that the “problem posing” was listed in the mathematics curriculum and became its important element. Nowadays, there is still the lack of a deep understanding of the nature of teaching and its basic problem such as how to deal with the relationship among problem posing, creating situations, problem solving and teaching goals, etc. As a result, a problem-posing approach is widely accepted by teachers, but few of them can really teach students to ask questions (Zhang 2002).

Chinese students generally do not have the learning experience of proposing mathematics problems and they have low awareness of problems. In order to change this situation, we conduct the research on mathematics teaching experiment of SCPBI in primary and secondary schools since 2001 according to the promulgation and implementation of national elementary education mathematics curriculum standard.

Experimental Objectives

First, the unbalanced phenomenon between “problem posing” and “problem solving” will be analyzed and resolved in mathematics education in Chinese primary and secondary schools. Second, teaching concepts and teaching behaviors that teachers are concerned with students’ problem-solving ability regardless of their ability to pose problems should be avoided. Third, students’ ability to coordinate the development on posing problems, understanding problems and solving problems from mathematics angle should be promoted. Meanwhile, through the teaching of “setting situations and posing problems” in primary and secondary schools, we will inspire students to learn mathematics, let them obtain mathematics experience and skills of “problem posing” so as to improve their ability to learn mathematics. As a result, “problem posing” can

be encouraged against the background of basic education curriculum reform in China, which contributes to the transfer from the curriculum idea into the teaching reality.

Experimental Contents and Measures

In view of Chinese mathematics education malpractice, the mathematics teaching model of “creating situations—posing problems—solving problems—applying mathematics”, called “situated creation and problem based learning” (SCPBI) is proposed.

Much attention has been paid to basic knowledge and basic mathematics skills instead of innovation consciousness and practical ability in mathematics education in Chinese primary and secondary school for quite a long time, which leads to the gradual emerging of contradiction and problems in mathematics learning, for example, students get accustomed to “learning to answer”, but they are not good at “learning to pose” (Huang, 2002). Their problem consciousness is gradually weakened (Zhu & Zhan, 2002) and their ability to propose problems become commonly low, too (Nie & Wang, 2000).

In order to change the situation in which many malpractices in the Chinese traditional mathematics education, such as students’ low ability to propose problems and attention paid to the result rather than the process, the research class directed by Lü and Prof. Wang (2000), have developed one mathematics teaching model—mathematics teaching model of SCPBI in primary and secondary schools (Figure 1) (Lü & Wang, 2001) . The model aims to train students’ innovative consciousness and practical ability.

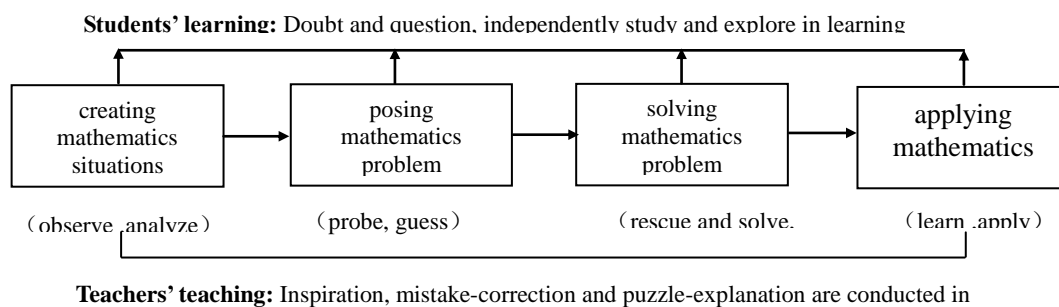


Figure 1. Mathematics teaching model of “SCPBI” in primary and secondary schools.

The basic goal of SCPBI is to train students’ ability to pose problems, and then promote students’ ability to coordinate the development on problem posing, problem understanding and problem solving from mathematics angle.

SCPBI has four basic links: creating situations, posing problem, solving problem and applying mathematics. Creating mathematics situations is the prerequisite. Mathematics problem posing is the core, while mathematics problem solving is the purpose, and mathematics application is the home for knowledge to return to.

As a matter of fact, SCPBI refers to a kind of mathematics teaching driven by “questions or problems.” In order to achieve its basic goal effectively, we regard stimulating students’ problem consciousness as the logical starting point and mainline. To be specific, the irritant mathematics material is provided to students through situations, students’ curiosity and knowledge-seeking desire can be stimulated. Through observing and exploring mathematics situations, students can find, pose and solve mathematical problems under the guidance of teachers.

Since 2001 the experimental study related to SCPBI has been conducted in some primary and secondary schools in Guizhou province, taking “using the experience of a selected spot(unit) to promote the work in the entire area, step-by-step promotion, rolling developing” as the guiding ideology. In order to reflect the differences of students’ mathematics thinking and learning ability in various geographic and economic situations, this experiment has been done in Guiyang (the capital city of Guizhou Province) and Xingyi (the main city of Southwest Buyi and Miao Autonomous Prefecture of Guizhou), selecting the first 6 experimental schools and later 11 experimental schools, including 5 primary schools and 1 secondary school in the downtown area of Guiyang, and 8 primary schools in the downtown of Xingyi, and 3 secondary schools in the suburb of Xingyi. The subjects range from the first grade pupil to the seventh grade students. There are 2,016 students in 34 experimental classes. 26 of them are in primary schools and 10 in secondary schools. With the deepening of the experimental study, the number of the experimental schools and classes is growing every year, and the area of the experiment is being widened.

Increasing and enhancing teachers’ knowledge and skills of how to teach “problem posing” and promoting the effective development of SCPBI experiment

How to improve mathematics teachers’ knowledge and technical skills of “problem posing”, especially in rural areas? This is related to teacher education, and is also a realistic problem that needs to be solved urgently in the process of putting mathematics curriculum standards into practice. In rural areas, the professional development space of mathematics teachers is generally narrow and small. Therefore, combined with SCPBI experiment carrying out in remote rural area of Guizhou, the *School—based teaching research based on interscholastic cooperation in some regions* is launched. (*ICR type school-based teaching research* for short). To be more exact, the research aims to improve experiment teachers’ knowledge and technical skills of “problem posing”. Specifically, under the guidance of experts, it is to take schools’ cooperation in some regions as the foundation, take SCPBI experimental study as the platform, and take class example as the research object. All mathematics teachers in the experimental schools study together with the mathematics backbone teachers in these schools, trying to master the ability as soon as possible (Xia & Lü, 2006).

Up till now, ICR type school-based teaching research has made teachers in rural areas aware of the meaning and method of “problem posing”, the essence of SCPBI, its method and strategy, the Assessment on the ability that students pose mathematics

problems, and caused them to obtain the related teaching knowledge and skills. Based on the extension of ICR type school-based teaching research in other areas, the contradiction needing specialized instruction is effectively alleviated. Therefore, it has promoted SCPBI experiment to be carried out effectively.

Teachers play the guiding role in teaching to promote the development of students' problem consciousness and probing ability

A teacher is the initiation, the maintenance or the promotion for students' study behaviors (Zhong, Cui, & Zhang, 2001). SCPBI can promote effectively the development of students' problem consciousness and probing ability, which depends upon playing the guiding role of teachers' full teaching. Therefore, in SCPBI, both students' main body status and teachers' instruction role should be emphasized. In classroom teaching, teachers can adopt all kinds of teaching methods, among which heuristics is the center. Meanwhile, relevant teaching strategies are chosen and used in a targeted manner, according to the experiment progress and the basic situation of students.

In the initial the SCPBI experiment period, encouraging students to pose questions is the basic teaching strategy. Students are taught to tell the difference between the questions, which is used to inspect whether they understand the true problem. Teachers help them clarify the constitution of "problem", know why people need to pose problems and why "problem" is important, and so on. Meanwhile, through the demonstrative inquiry, teachers enable students to master basic skills and methods of "problem posing" in observing and imitating, so as to enhance their self-confidence in learning mathematics in mathematics teaching activities of posing problems.

In the mid and later SCPBI experiment period, according to students' different situations, teachers use flexible teaching strategies including teachers' leading and students' self-learning. The strategies of teachers' leading role are as follows: setting up the mathematics situation, leading to problem posing, discussing and exchanging ideas, promoting cooperative study, paying attention to problem solving and mathematics application, assigning situational tasks, carrying out mathematics activities, attaching importance to intensive lecture, guiding students thinking, paying attention to reviewing and summarizing, and developing students' meta-cognition. The strategies of students' self-learning consist of going deep into situations, observing carefully, extracting mathematics information, pondering earnestly, questioning boldly, posing mathematics problems, interactive probing in cooperation, resolving the mathematics problem actively, developing special study and topic study, exercising self-examination profoundly and reviewing and arranging knowledge systematization and modularization (Yang & Wang, 2004).

Basic Experimental Study Results

Experiment has stimulated students' interest in studying mathematics

Arousing students' interest in learning mathematics helps to ease their mathema-

tics anxiety, and enhance their confidence and self-awareness in probing into problems (Li & Lü, 2004). In order to test how much SCPBI can affect students' interest in mathematics, we chose 327 students from B junior high School, S junior high School and Z junior high School in Xingyi in 2003. Among them there are 101 from experimental classes while 226 from control classes. By questionnaire, a comparative study is made on students' interest in mathematics in both experimental and control classes. (Table 1)

Table 1
**Percentage of Students' Interest in Mathematics
in Experimental and Control Classes**

Students	<i>N</i>	Percentage of students interested in mathematics	<i>Z</i>
Experimental class	101	84.1%	
Control class	226	60.2%	4.35

As the table shows, 84.1 percent of the students from experimental classes have expressed their interest in mathematics (only 72.5 percent of them shared the same views in the investigation before SCPBI experiment). But the students' proportion was only 60.2 percent in control classes. After the double overall rate of hypothesis testing, it is easy to see that SCPBI has very remarkable effects on students' interest in learning math. ($|Z| = 4.35 > 2.58$, $p < 0.01$)

Students' interest in mathematics (divided into "interested" and "uninterested") and its relative ratio are subject to binomial distribution, and $\min(np, nq) = \min(73.28) = 28 > 10$. Therefore, at the same time, we also carried on the single overall rate of hypothesis testing on experimental class students' interest in learning mathematics. The results also show that SCPBI has extraordinarily notable effects on students' interest in learning math. ($|Z| = 2.64 > 2.58$, $p < 0.01$)

Experiments have enhanced the students' abilities to pose problems

"Problem posing" is the core of SCPBI and improving students' abilities to pose problems is the basic goal. This research tends to make SCPBI experiment. After two years' experimental study, a comparative test is conducted on students' ability to pose problems. All the subjects are from 9th-grade of F Secondary School in Guiyang and of S Secondary School in Xingyi. (Experimental School I and Experimental School II for short) (Lü & Wang, 2006).

Two different types of mathematics situations were provided to the students in the test— taking both algebra content and geometry content as the situations, the students were requested to pose as many mathematics problems as possible according to the situations.

Algebra situation: Mother gives Xiaohong 20 Yuan, letting her buy study materials. In the store, the price of a notebook is 3 Yuan and the price of a ball pen is 2 Yuan.....

Geometry situation: Connect each side midpoint of a regular triangle, form one

new regular triangle and reciprocate this process (Figure 2).

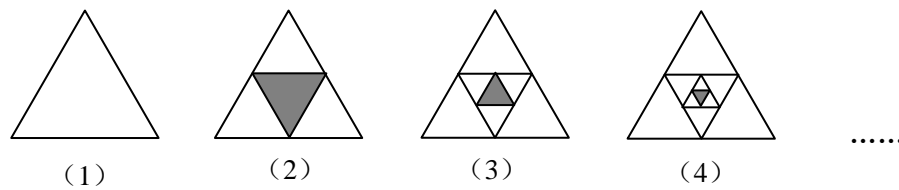


Figure 2. Connecting each side midpoint of a regular triangle.

Based on the analysis of students' mathematical responses in the task of posing problems, the types of the problems posed by students are sorted out. (see Table 2)

Table 2

Students' Typical Mathematical Responses in the Task of Posing Problems

Problem type	Problems posed according to algebra situation	Problems posed according to geometry situation
1st type	Xiaohong bought 5 notebooks, also bought 5 ball pens. (General statement)	How much is the shadow partial area in chart (2). (Unsolvable mathematics problem)
2nd type	How much does Xiaohong pay to buy 6 ball pens? (Simple mathematics problem)	How many triangles are there in chart (2)? (Simple mathematics problem)
3rd type	If more than five notebooks are bought, the price will be 90% of the initial cost. How many notebooks can Xiaohong buy if having 20 yuan mostly? (Development problem)	When carrying on n times, what is the ratio between shadow partial perimeter and regular triangle in chart (1) perimeter? (Inquiring problem)

Meanwhile, the students' ability to pose problems can be divided into five levels:

Level I: More than six different mathematical problems are proposed, in which two problems belong to the 3rd type at least.

Level II: More than six different mathematical problems are proposed, in which one problem belongs to the 3rd type and one belongs to the 2nd or the 1st type at least.

Level III: More than four different mathematical problems are proposed, in which two problems belong to the 2nd type or one belongs to the 2nd type and 1st type, one belongs to the 1st type at least.

Level IV: More than two different mathematical problems are proposed, in which one or two problems belong to the 1st type at least.

Level V: No mathematical problems are proposed.

Levels on problems proposed by students in the experimental class and the control class are summarized in Table 3. It can be seen that about 70 percent of the students in experiment class is on the 1st level, significantly better than those of the control class. It shows that SCPBI has extraordinarily notable effects on students' ability to pose problems.

Table 3
Levels on Problems Proposed by Students in the Experimental Class and the Control Class are Summarized

School	Class	Levels on problems proposed				
		Level I	Level II	Level III	Level IV	Level V
Experimental class (1st)	Experimental class n = 47	74.47%	8.51%	17.02%	0	0
	Control class n = 32	25%	37.5%	37.5%	0	0
Experimental class (2nd)	Experimental Class n = 69	68.12%	17.39%	14.49%	0	0
	Control class n = 65	14.29%	20%	60%	5.71%	0

Promoting students' ability to learn mathematics development

To check the SCPBI functions on students' ability to learn mathematics, we established one test which conformed to the national mathematics curriculum standard. The comparative investigation and analysis on 9th-grade Students who have been experimented in the experimental class and the control class of Secondary School for two years, including 61 students in the experimental class, and 57 in the control class. On the principle of inspecting students' basic knowledge and attaching importance to examining the students' ability, the test papers add some items of mathematics application and capability, and simultaneously foundational, comprehensive and practical natures are also given dual attention.

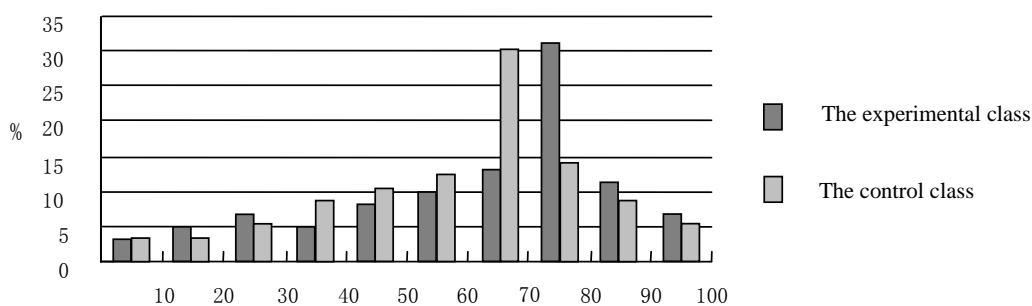


Figure 3. The distribution of the test results.

The distribution of the test results is revealed in Figure 3. It can be seen that the two distribution pictures are approximately in similarly normal distribution. Exam pass rate: 62.3 percent in the experimental class; only 56.1 percent in the control class. Outstanding rate: 18 percent in the experimental class; 14 percent in the control class.

The highest score: 96 in the experimental class; 91 in the control class. The main scores range from 70 to 80 in the experimental class, while the main scores in the control class range from 60 to 70.

The test is carried out in the experimental class and the control class, and its results show that SCPBI has extraordinarily notable effects on students' ability to learn mathematics. ($|Z| = 2.06 > 1.96$, $p < 0.05$)

Conclusion and Discussion

This research indicates that SCPBI has significant effects on students' interest in learning math, student's ability to pose problems and to learn mathematics. From the author's point of view, the reasons producing these effects are mainly as follows:

First, the classroom instruction environment that is driven by "problems" is provided by SCPBI, promoting the students development on problem consciousness and probing into mathematics ability. Furthermore, through creating situations and teaching guidance, teachers let students not only propose their own questions, but also construct their own problems, including simple questions and exploratory problems. Students are required to solve not only well-structured problems but also ill-structured problems. And problem posing is throughout the process of finding the solution to the problem.

Second, in the process of SCPBI experiment, specialized instruction and ICR type school-based teaching research have changed teaching concepts and behaviors that focused on developing students' ability to solve problems regardless of their ability to pose problems. They also improved mathematics teachers' teaching knowledge and technical skills of "problem posing" effectively. They not only approach mathematics problems, how to "teach" students to solve problems, but also "learn" how to teach students to pose problems. This has provided the essential condition for the effective development of SCPBI experiment.

Third, teachers choose and use relevant teaching methods and strategies in a targeted manner. They play an effective role in guiding and promoting the formation of the classroom instruction environment that is driven by "problems", which plays a key role in achieving SCPBI basic goals effectively.

The research takes training students' problem consciousness and the ability to pose problems as a basis, bringing forward the pattern of mathematics teaching model—SCPBI model. Its teaching aim is to train students' innovative consciousness and practical ability. In the process of the experiment, effective approaches and methods of improving teachers' teaching knowledge and the technical skills in "problem posing" are probed, and how to choose and use the teaching methods and tactics of "problem posing" for teachers is also discussed and experimented. As a result, the contradiction between "problem posing" and "problem solving", which has existed in basic mathematics education in China, can be solved practically. And the "problem posing" and regular teaching can be coordinated appropriately. Meanwhile, effective approaches and methods can be offered with the transfer of "problem posing" from the

curriculum idea to the teaching reality.

At present, SCPBI experimental study has been expanded gradually from Guizhou province to such places as Yunnan, Sichuan, Chongqing, Zhejiang, Guangdong and other provinces or cities. Up to now, there have been more than 1000 experimental classes and over 600 experimental schools, in which all the subjects are from 1st-grade to 12th-grade. At the same time, SCPBI is also applied to the teaching of physics, geography and other courses in primary and secondary schools, receiving good assessment. This not only represents the vigor embedded in SCPBI pattern, but also promotes building a classroom-teaching culture driven by “problems” against the background of China’s fundamental curriculum education reform.

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