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Using a Structured Instrument to Improve the Mathematical Confidence Levels of Female College Students

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The study examined the effectiveness of a researcher-developed structured problem solving instrument on 21 female college students' mathematical confidence. The structured instrument, Examine-Strategize-Solve-Reflect (ESSR), was designed to be an assessment instrument as well as an instructional tool. Participants took a series of 8 quizzes utilizing the ESSR instrument to solve mathematical problems they had never seen before. Prior to and after using the ESSR instrument, the participants took a mathematical confidence survey. Paired-samples t-tests indicated that students' overall confidence increased significantly over the 3-month period. Further crosstabs analyses indicated that the largest effect sizes could be directly tied to the structure of the ESSR instrument.

Key words: mathematical problem solving, mathematical confidence, structured instrument, female college students, assessment.

Introduction

Female students tend to have higher levels of mathematics anxiety than male students (Miller & Bichsel, 2007) and high levels of mathematics anxiety have been found to relate to one's confidence in their ability to do mathematics and, at the same time, negatively influence one's problem solving abilities. In addition, students' confidence regarding their ability to do mathematics has been found to strongly influence their problem solving ability (Schoenfeld 1983a, 1985), thus helping students become successful problem solvers may increase their overall confidence in their ability to perform certain mathematical tasks.

The purpose of this study was to help female college students become better problem-solvers, thereby enhancing their mathematical confidence. To this end, the study examined the effectiveness of a researcher-developed structured problem solving instrument on students' mathematical confidence.

This structured instrument not only measured problem solving skills, but also served as an instructional tool. The structured instrument, Examine-Strategize-Solve-Reflect (ESSR), adapted from Wuttke and Wolf's (2007) Measurement and Assessment of Problem Solving (MAPS) instrument, was designed in such a way that mathematical problem solving ability could be easily measured while at the same time support could be provided to guide students through the problem solving process. The study was conducted at a women's college in Southern California and therefore only female participants could be recruited. The creation of this structured assessment instrument not only allows mathematics instructors to evaluate female college students' problem solving abilities, but also presents a method of problem solving instruction, hopefully providing a meaningful way for female college students to enhance their problem solving abilities, as well as their mathematical confidence.

The study addressed four research questions in order to determine the strengths and weaknesses in female college students' mathematical problems solving, the effectiveness of the ESSR instrument in measuring and teaching mathematical problem solving, and whether mathematical confidence increased significantly or not. However, this paper only addresses the following research question: How effective is the ESSR instrument in increasing female students' confidence in their mathematical abilities? Further studies that include both males and females need to be performed in order to see if the ESSR instrument is effective in increasing all students' confidence.

Literature Review

The literature indicates that mathematical confidence is the best predictor of mathematical problem solving success, more so than any other variable, even mathematics anxiety and gender (Pajares & Miller, 1995). Ryan and Ryan (2005) integrated stereotype threat research with achievement goal research to develop a model that could help educators better understand how stereotype threat affects students' mathematical confidence and performance in mathematics. Their theory asserts that students in high stereotype conditions, such as females who believe that males are better at mathematics, will not perform as well on mathematics tests and will experience a decrease in their mathematical confidence.

There are several models of mathematical problem solving that can be used when working with pre-college and college aged students: Pólya's

(1945), Schoenfeld's (1992), and Mason, Burton, and Stacey's (1982) to name a few. All of the models include four basic steps: (1) think about the problem, what is given, what is being sought, and what mathematics is related to the situation; (2) strategize on different ways to solve the problem, (3) solve the problem with one or more specific strategies; and (4) consider the solution to see if it is reasonable and whether there might be a better way to solve that type of problem in the future. Many students skip step one and jump right in to solve the problem, without first thinking about the problem. When they do this, mistakes are frequently made. Teachers can help reinforce the steps, especially step one, as they model solving various types of problems in class. The structure of the ESSR instrument forces students to work through each of the steps associated with mathematical problem solving.

For Skolnick, Langbort, and Day (1982), problem solving was at the center of their approach to help females build competence and confidence in their mathematical abilities. They link mathematical confidence to problem solving skills, since expert problem solvers have the belief in themselves that they can do it. The authors provide many examples and activities to be used with females in order to improve their mathematical confidence. The study described in this paper also uses mathematical problem solving as a means for improving the mathematical confidence of female college students.

After reviewing the literature on mathematical problem solving, the question remains, how can mathematics instructors help increase their students' mathematical confidence? The literature reviewed suggests there are several aspects teachers can focus on: modeling good problem solving skills, reducing stereotype threat, reducing mathematics anxiety levels, and using an instrument that informs instruction on mathematical problem solving (Bandura, Barbaranelli, Vittorio Caprara, & Pastorelli, 2001; Curtain-Phillips, 1999; Schoenfeld, 1983b). It is vital that a student believes she has the ability to solve mathematical problems in order for that student to be successful (Bandura et al., 2001). Thus if teachers can boost their students' confidence, their students will have a better chance of succeeding in mathematics.

The researcher-developed Examine-Strategize-Solve-Reflect (ESSR) instrument (Appendix A) was structured so that it asks questions in a logical order and walks students through the steps associated with mathematical problem solving. This will help the students to learn the important steps in mathematical problem solving and will help the evaluator be able to see what students are thinking as they solve a mathematical problem.

Method

Participants

The students recruited as participants for this research study were conveniently sampled from students in the researcher's Mathematical Analysis for Business course in the Fall semester, 2009 at a small, independent liberal arts college, primarily for women, located in a west coastal urban city. According to the college's website, its mission is to offer a dynamic learning experience in liberal arts and sciences to a diverse student body. The college is also dedicated to providing an excellent education enhanced by an emphasis on building leadership skills and fostering a spirit to serve others. The topics covered in the Mathematical Analysis for Business course included: solutions of systems of equations and inequalities, exponential and logarithmic functions, linear programming, and mathematics of finance. Emphasis was placed on the application of mathematics to problems in business.

The Fall 2009's Mathematical Analysis for Business class consisted of 23 female college students, of which 21 agreed to participate in the study. Most of the participants were business majors (83%) and their ages ranged from 18 to 27, with most (75%) of the students being 18 to 21 years old. The ethnic breakdown of the participants was as follows: 79% Hispanic or Latino, 8% White, 8% Asian, and 4% African-American. In terms of the participants' previous mathematics courses, 56% had taken Algebra I in high school, 74% hade taken a Geometry course in high school, 83% had taken Algebra II in high school, and 74% of the participants had taken a statistics course in either high school or college.

Mathematical Confidence Survey

The Mathematical Confidence Survey was created in 2008, by the researcher, to measure how confident college students are in their ability to successfully solve mathematical problems. The survey contains 13 items that students rate according to their confidence level (no confidence, very little confidence, some confidence, much confidence, complete confidence) with respect to the described situation. Examples of such situations are using algebra to solve a problem, solving a problem with or without a calculator, and solving a math problem they have never seen before. The survey was created by the researcher after twenty years as a mathematics educator and by pondering the various ways students could solve problems as well as the various situations in which students might solve mathematical problems (such as individually or in a group). To score the survey, students' rankings are averaged to calculate an overall confidence score between 1 and 5, with 1 indicating no mathematical confidence and 5 indicating complete mathematical confidence.

The Mathematical Confidence Survey was piloted twice with two different samples of college students. The survey originally had only ten items; after the first pilot study, three more items were added. The first pilot study, in 2008, involved ninety-nine male and female students from a local community college who took the Mathematical Confidence survey. Reliability tests were conducted on the data collected during the 2008 pilot study. Overall the ten confidence statements were found to be reliable with a Cronbach Alpha of .874.

In 2009, another pilot study of the Mathematical Confidence Survey was conducted with 14 college students in a summer school class held at the same location as the current study. The survey was given twice, once at the beginning of the six-week course, and then again at the end of the course. Overall the thirteen confidence statements given as a pretest were found to be reliable with a Cronbach Alpha of .886 and also when given as a posttest (Cronbach alpha = .899).

Design and Procedure

In order to see if there was a relationship between becoming better problem solvers and increasing mathematical confidence, a mathematical confidence survey was used along with the structured mathematical problem solving assessment instrument, Examine-Strategize-Solve-Reflect (ESSR). The confidence survey was used to establish baseline mathematical confidence and the ESSR instrument was used (via two pretests) to establish baseline problem solving skills of the participants. The survey asked students to rate how confident they felt in performing various mathematical tasks, such as using geometry to solve a mathematics problem. Throughout the semester, participants took six more quizzes using the ESSR instrument. In addition, four interventions were conducted. Each intervention focused on one of the four steps of mathematical problem solving to provide techniques and suggestions for improving in that area. Pedagogies such as cooperative learning, questioning strategy, and guided practice were incorporated into the interventions. At the end of the semester, students again took the mathematical confidence survey.

To answer the research question: *Is the ESSR instrument effective in increasing female college students' mathematical confidence?* a paired-samples t-test was conducted to find out whether the pre-confidence scores significantly differed from the post-confidence scores. This was done for students' overall confidence scores and for each individual confidence item, 13 in all. In addition, a crosstabs analysis was used to calculate Kendall's tau-b in order to examine the association between confidence item 13 (using a structured instrument to solve mathematical problems) and the other 12 confidence items. Effect sizes were also calculated to indicate the relative strength of any significant differences between the two surveys.

Results

To calculate a participant's confidence score, 13 individual confidence scores were averaged to calculate an overall confidence mean. This was done for both the pretest and the posttest, and then a paired samples t-test was run using SPSS to find the difference. The average for the pre-confidence means was 3.68 with a standard deviation of 0.54 and the average for the post-confidence means was 4.28 with a standard deviation of 0.44. The mean difference of -0.59 was considered statistically significant (t (20) = -4.90, $p <$ .05 (two-tailed), d = 1.21). This indicates that the post-confidence means were significantly higher than the pre-confidence means. The effect size of the difference was 0.52.

A paired samples t-test was also run for each of the 13 items on the Confidence Survey in order to check if the difference between the students' pre-test ratings and the post-test ratings were significant or not. Eight of the 13 confidence items were determined to be significantly higher after taking a course which included use of the ESSR instrument, indicating participants' confidence on those items increased significantly after participating in the study.

Students' ratings on the post-confidence surveys were higher in all 13 areas compared to their ratings on the pre-confidence surveys; however the differences were not all statistically significant. Table 1 summarizes the results of the 13 paired-sample t-tests run on the 13 confidence items.

Table 1

Pre and Post Mean Confidence Ratings

Item Number	Pre-Confidence Mean	Post-Confidence Mean	Difference	Effect Size (Cohen's *d*)
1	4.14	4.48	-0.34*	0.441
2	4.29	4.76	-0.47*	0.741
3	3.81	4.10	-0.29	0.367
4	3.00	3.48	-0.48*	0.630
5	4.19	4.48	-0.29	0.452
6	3.71	4.10	-0.39	0.466
7	3.43	4.57	-1.14*	1.463
8	4.00	4.33	-0.33	0.390
9	4.29	4.48	-0.19	0.238
10	2.48	3.57	-1.09*	1.211
11	3.38	4.33	-0.95*	1.236
12	3.43	4.29	-0.86*	1.124
13	3.71	4.33	-0.62*	0.819

* indicates difference is significant at the .05 level

It was not surprising that item 7 (determining what the problem was asking) was ranked statistically significantly higher on the post-survey than on the pre-survey, since the ESSR instrument instructed students how to examine a problem by circling the keywords in a problem in order to better understand the mathematical situation. A lack of significance for item 8 (verify or check the results of my answer to see if it seems reasonable) was also not surprising. On the ESSR pretests participants were found to be weakest at checking their solutions to mathematical problems. The lack of significant increase in participants' confidence levels on item 8 indicates students are aware of their difficulty with this step and even though their confidence levels in their ability did increase after using the ESSR instrument repeatedly, the increase was not statistically significant. Furthermore, for item 9 (solve a math problem on my own if I have seen the teacher solve a similar one in class), it is understandable why the ESSR instrument did not improve students' confidence in this area. In order to get a true measure of mathematical problem solving, the problems selected for the pre and post ESSRs were specifically chosen because the students had not previously seen or worked with problems of that type. The researcher wanted to see if students could use the four problem-solving steps to examine, strategize, and solve a never-before-seen mathematical problem.

The last four confidence items (10, 11, 12, and 13) were all found to be ranked statistically significantly higher on the post-confidence survey and all four items were addressed by utilization of the ESSR instrument. The mathematical problems that students were presented with were all ones they had not seen before and many could be solved by making a table or drawing a picture. In addition, the ESSR instrument encouraged participants to think of as many different strategies as possible with which to solve the given mathematical problems; it is therefore not surprising that students in the study felt significantly more confident that they could solve a mathematics problem using different strategies than they did at the beginning of the study. Finally, the ESSR instrument was a structured instrument, which walked students through a series of questions designed to help them understand and solve a mathematical problem. It was expected that participants in the study would rank their confidence in solving a mathematics problem using a structured method significantly higher on the post-survey, and they did.

Although participants' mathematical confidence did increase in every one of the 13 items, the largest effects were in five of the items: identify the unknown, solve a math problem I have never seen before, solve a math problem by sketching graphs or diagrams, solve a math problem by using different strategies, and solve a math problem using structured methods. The five items with the largest effect size can directly be tied to the ESSR instrument and the study that was conducted. Item 7 (identify the unknown, i.e., determine what the problem is asking me to find) corresponds directly to one of the students' noted strengths under the Examine component: student is able to correctly identify what the problem is asking them to find. Students began the study with this as one of their strengths and with repeated practice offered by the ESSR instrument, students' confidence in this ability increased. Items 10 (solve a math problem I have never seen before) and 11 (solve a math problem by sketching graphs or diagrams) were also integral to the study. Problems for the ESSR instrument were purposely chosen because they were problems students had never seen before and the participants were told this throughout the study. In addition, one of the interventions focused on various strategies that could be selected to solve a mathematical problem. Drawing pictures and diagrams was one of the highlighted strategies that we discussed at length. Students felt more confident about their abilities in these two areas as a direct result of the study.

Item 12 (solve a math problem by using different strategies) is directly related to one of the sub-questions of the ESSR instrument: student proposes a variety of applicable and appropriate strategies. Students learned from the using the ESSR instrument and from the Strategize intervention that all problems can be solved by a variety of methods. They also practiced this technique as part of the Strategize intervention. It is therefore not surprising that their confidence increased significantly in this area. Item 13 (solve a math problem using structured methods) is the main idea behind the ESSR instrument. The structure of the instrument forced them to go through each of the four steps to solve a mathematical problem. The participants' confidence in their ability to follow and use such an instrument increased with repeated use of the structured ESSR instrument. Indeed the mathematics course that utilized the ESSR instrument, along with the four interventions, was effective in increasing female college students' mathematical confidence.

To investigate the relationship between Confidence Item 13 (solve a math problem using structured methods) and the other 12 confidence items, Kendall's tau-b was calculated, treating the students' rankings from one to five as ordinal values. The analysis indicated a significant positive relationship between students' confidence using a structured method (item 13) and seven of the other 12 confidence items (see Table 2). The seven items correlated to item 13 were: use algebra to solve a mathematical problem, solve a mathematical problem without using a calculator, use geometry to solve a mathematical problem, solve a math problem on my own if I have seen the teacher solve a similar one in class, solve a math problem I have never seen before, solve a math problem by sketching graphs or diagrams, and solve a math problem by using different strategies.

Table 2

Relationship to Using Structured Instrument Results

Confidence Item Related to Item 13 (using structured instrument)	Kendall's tau-*b*	Significance
Use algebra to solve a mathematical problem	2.662	.008*
Use a calculator to solve a mathematical problem	1.751	.080
Solve a mathematical problem without using a calculator	5.457	.000*
Use geometry to solve a mathematical problem	3.856	.000*

Solve a mathematical problem with the help of other students	0.691	.489
Use trial and error to solve a mathematical problem	1.788	.074
Identify the unknown (i.e. determine what the problem is asking me to find)	1.057	.291
Verify or check the results of my answer to see if it seems reasonable	1.604	.109
Solve a math problem on my own if I have seen the teacher solve a similar one in class	2.237	.025*
Solve a math problem I have never seen before	5.424	.000*
Solve a math problem by sketching graphs or diagrams	4.878	.000*
Solve a math problem by using different strategies	2.711	.007*

* Indicates significant at the .05 level

The seven significant positive relationships indicate that when students used the structured instrument to solve mathematical problems they were highly confident in their abilities to use algebra, solve a problem without using a calculator, solve problems they have seen before and ones they have not, use sketches to solve a problem, and/or use different strategies to solve a problem. The connection between the uses of a structured instrument to so many other confidence items, suggests that the structure of the instrument played an important role in increasing students' confidence in the other items. Female college students in this study indicated that they were more confident in using structured methods to help them solve mathematical problems and the structure of the instrument was highly correlated with many other items about which they felt more confident. This provides evidence that the structured instrument and course together led to increased confidence among the students in the study. The ESSR instrument shows promise as a method for supporting female students in increasing their confidence; however, further studies are needed to verify this claim.

Discussion and Conclusion

The research results indicate that the ESSR instrument is an effective method of increasing female college students' mathematical confidence.

Female college students in this study ranked themselves significantly more confident in their mathematical problem solving ability at the end of the study than they did at the beginning of the study. Furthermore, the largest effect sizes were in five confidence items that can be tied directly to the structure of the ESSR instrument: identifying the unknown, solving a math problem I have never seen before, solving a math problem by sketching graphs or diagrams, solving a math problem by using different strategies, and solving a math problem using structured methods. Studies involving comparison groups and or a control group are recommended in order to prove that the structure of the ESSR instrument is indeed the reason for the increased levels of mathematical confidence.

This study incorporated mathematical problem solving as the means to increase female college students' mathematical confidence. The results support Skolnick et al.'s (1982) belief that problem solving is the best way to help female students build competence and confidence in their mathematical abilities. In addition, the structure of the instrument modeled for the participants what good problem solving should look like, just as Schoenfeld (1983b) suggests, and the basic steps of mathematical problem solving were emphasized and reinforced by repeated use of the ESSR instrument. Participants gained confidence in their abilities to perform mathematical tasks after repeated use of the structured ESSR instrument.

Both the ESSR instrument and the interventions emphasized ways in which the students could make sure they understood the problem, specifically identifying the unknown or goal of a problem, and focused on looking at more than one way (i.e. strategy) to solve the problem. In addition, problems the students had never seen before were purposely chosen for the ESSR quizzes in order to see how the students reacted to new mathematical problems. Data analyses indicated that participants' confidence levels in the above areas was significantly tied to their confidence in using a structured instrument. Thus the conclusion is that the structure of the ESSR instrument was a key factor in increasing the participants' mathematical confidence.

This study provides college mathematics instructors with a structured instrument, ESSR, which can be used, not only to assess female college students' problem solving strengths and weaknesses, but also to instruct students in the processes and strategies necessary for successful mathematical problem solving. The methods of this study also provide a means to help female college students increase their mathematical confidence. However, there are some limitations to this study. Without a comparison group or control group, it is hard to generalize the results of this study to all college students. Furthermore, since this study was embedded within a mathematics course, it is difficult to know if the ESSR instrument was the cause of students' increased mathematical confidence levels or if the combination of the course and the ESSR instrument, or just the course itself was the cause(s). Further studies need to be conducted in order to verify that it was the structure and repeated use of the ESSR instrument that led to increased mathematical confidence levels.

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Appendix

ESSR Instrument

Examine-Strategize-Solve-Reflect (ESSR) Instrument

Problem: {different word problem to be inserted each time}

Questions:

Examine Initial Situation: For the problem above, please underline or circle the keywords that are important to understanding the problem. Are there any hidden conditions or assumptions that might help you understand the problem? What is the problem asking you to find?	
Strategize: Look at the list of strategies on the back of this page. What are some different ways you could solve the problem? List as many as possible. Please explain why you think these methods are appropriate for this particular	***Solve:*** Please solve the problem using the method you decided upon. Then check your work (show your steps and explain your reasoning).

problem. Decide which method/strategy you will use.	

Reflect:
Did you achieve the goal of the problem? How do you know?
What errors or mistakes did you make as you were solving the problem? If no errors were made in your solving process, identify the key steps that might confuse other students.
Did you choose the most appropriate method/strategy?
Now that you have solved the problem, do you see an easier method/strategy that you did not think of before?
What did you learn from doing this problem and how can it help you solve future problems?

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