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Exploring Tertiary Teachers' Promotion of Caring Communities in Mathematics

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In this article, we utilize a networking of theories approach to connect the documentational approach to didactics (DAD) and a theory of meanings to explore and develop models of two tertiary teachers' understandings of developmental mathematics and its students, and the resources each teacher utilizes to support instruction. We illustrate how teachers' understandings related to developmental mathematics and its students promote and support mathematical caring relations and a sense of belonging as learners of and contributors to mathematics in their students. In addition, we discuss the affective aspects of the schemes of meanings that came to each teachers' minds as a result of these understandings, including anxiety, frustration, interest, identity, motivation, persistence, and self-efficacy. We posit that participating teachers' meanings demonstrate their attention to decentering; that is, to attempt to imagine one's experience from another perspective. Finally, we describe how networking the documentational approach and a theory of meanings provide researchers and mathematics teacher educators with a framework focused on teachers' documentation work that supports identification of the schemes of meanings with which teachers teach and has the potential to support the design of productive professional learning experiences (to student learning) for both preservice and inservice teachers.

Keywords: caring communities, sense of belonging, affect, documentational approach to didactics, theory of meanings

In the United States, college and university policies typically require new students to be assessed in mathematics through some form of placement test. These tests are commonly designed and evaluated by a third-party (e.g., ACCUPLACER, ALEKS, and COMPASS), although some institutions utilize placement tests developed by faculty in the institution's mathematics department (e.g., University of Wisconsin System). Students assessed as needing mathematics remediation are required to complete remedial mathematics courses prior to taking college-level mathematics courses.

The need for remediation in mathematics is not uncommon for students at both two- and four-year public institutions. Chen's (2016) analysis of beginning tertiary students' course taking between 2003 and 2009 found 59.3% of students who began their tertiary education at a two-year public institution and 32.6% of students who began their tertiary education at a four-year public intuition, took one or more remedial (or developmental) mathematics course. Furthermore, Chen found that participation in remediation was more common among students of color and students from low-income backgrounds at both types of institutions. Unfortunately, a large percentage of students enrolled in developmental courses fail to pass them. According to Chen, 20% of remedial mathematics course takers beginning at both public two- and four-year institutions did not complete any of the developmental mathematics courses they attempted.

Although many consider tertiary remediation a United States phenomenon, research on remediation outside of the United States has increased significantly in the past decade, including Germany (Büchele, 2018), Hungry (Baranyi & Molonty, 2020), Italy (De Paola & Scoppa, 2014), Japan (McVeigh, 2015), Lebanon (Aoude, 2013), Taiwan (Yang et al., 2014), and the United Kingdom (Gallimore & Stewart, 2014). Recent research predicts that K-12 students, on average, could learn two months up to a full year less mathematics in 2020-21 due to the coronavirus pandemic compared to what they would learn in a typical year (Nagel, 2021; Sawchuk & Sparks, 2020). This global loss to learning implies a need for remediation in higher education for years to come (Azevedo et al., 2021). Related to this need, in this article, we report on a study that examined tertiary teachers' understandings of developmental mathematics and its students and the resources each teacher utilized to support instruction. Specifically, the study focused on teaching and learning developmental mathematics at the tertiary level and addressed the following research questions:

1. How do tertiary mathematics teachers understand developmental mathematics?
2. How do tertiary mathematics teachers understand the students enrolled in developmental mathematics?
3. How do tertiary mathematics teachers understand the resources they utilize to support developmental mathematics instruction?

Literature Review

There exists substantial research documenting the characteristics (e.g., academic preparedness, age, ethnicity, patterns of course taking, prior achievement, and socioeconomic status) of higher education developmental mathematics students that relate to retention and success (Benken et al., 2015; Boatman & Long, 2018; Cafarella, 2014; Goldrick-Rab, 2007; Mesa, 2012; Stigler et al., 2010; Waycaster, 2001). In addition, extensive research on efforts to reform developmental mathematics in higher education have focused on restructuring coursework, reforming developmental assessments and placement, redesigning curricula, and enhancing student support (Bailey et al., 2010; Castleman & Meyer, 2016; Fong & Melguizo, 2017; Henson et al., 2017; Jaggars et al., 2014; Ngo et al., 2018; Pheatt et al., 2016; Scott-Clayton et al., 2014; Visher et al., 2017; Weisburst et al., 2017; Yamada & Byrk, 2015; Zachry Rutschow & Mayer, 2018). More recent reform efforts attend to student learning through improved pedagogy (Bickerstaff et al., 2019; Cormier & Bickerstaff, 2020). Missing from this research are investigations into those at the forefront of attempts to implement such reforms—the instructors of developmental mathematics courses.

In this article, we report on a study that addressed Mesa et al.'s (2014) call for mathematics education research focused on issues of mathematics learning in community college and developmental contexts and collaboration with practitioners who have expertise in the teaching and learning of mathematics in such settings. As such, the study examined tertiary teachers' understandings of developmental mathematics and its students. Following Thompson and Harel (e.g., Thompson et al., 2014), we conceptualized *understanding* in terms of Piaget's notions of scheme and assimilation. For example, how a teacher *understands* her students enrolled in developmental mathematics is constituted by the meanings that come to that teacher's mind upon encountering situations involving developmental mathematics and its students.

Theoretical Framework

The study utilized a networking of theories approach (Bikner-Ahsbahs & Prediger, 2010) to connect frameworks and explore the research questions; specifically, we "network" the documentational approach to didactics (Gueudet & Trouche, 2009) and Thompson and Harel's (Harel, 2021; Thompson et al., 2014) theory of meanings.

The Documentational Approach to Didactics

The documentational approach to didactics (DAD) analyses "teachers' work through the lens of 'resources' for and in teaching: what they prepare for supporting their classroom practices, and what is continuously renewed by/in these practices"(Trouche et al., 2018, pp. 1-2). In the documentational approach, resource is grounded in Adler's (2000) work, which defines a resource as anything likely to 're-source', or "to source again or differently" (p. 207) the teacher's work. That is, all the "resources that are developed and used by teachers and pupils in their interaction with mathematics in/for teaching and learning, inside and outside the classroom" (Pepin & Gueudet, 2020, pp. 172-173). Such resources include text (e.g., textbooks, worksheets, and tests) and other material resources (e.g., calculators); digital-/ICT-based resources (e.g., online textbooks, Desmos, and GeoGebra); discussions between teachers, orally or online; students' written work; and teachers' discussions with mathematics teacher educators (Pepin & Gueudet, 2020). Teachers interact with resources, select them and work on and with them (e.g., adapting, modifying, and reorganizing) within processes where design and enacting are intertwined. Furthermore, teachers are introduced to and interact with resources through discussions with colleagues, professional learning experiences and via focused or random online searches. For Trouche et al. (2018), the expression *documentation work* encompasses all these interactions.

Central to the documentational approach is *documentational genesis*, which comprises two interrelated processes (Pepin et al., 2013): the process of *instrumentalization*, where a teacher's knowledge guides the choices she makes among various resources and the way these resources are appropriated; and the process of *instrumentation*, where the features of the resource or set of resources impact the teacher's knowledge she develops as a result of her interactions with the resource or set of resources. The process of documentational genesis results in the development of a *document* and can be represented by the equation (Pepin et al., 2013): Document = Resource(s) + Utilization Scheme. The concept of *scheme* is central in the documentational approach. According to Vergnaud (2013), schemes are "stable forms for organizing activity" (p. 57) and can be viewed as a structure organizing a subject's activity with a resource or set of resources for a given goal.

Thompson and Harel's Theory of Meanings

Thompson and Harel's theory of meanings (Harel, 2021; Thompson et al., 2014) is based on Piaget's notion of assimilation to a scheme and focuses on teachers' (schemes of) meanings, where a scheme is defined as "an organization of actions, operations, images, or schemes—which can have many entry points that trigger action—and anticipations of outcomes of the organization's activity" (Thompson et al., 2014, p. 11). For example, a "person's scheme for constant rate of change . . . could call upon her schemes for variation, accumulation, and proportionality" (Thompson & Harel, 2021, p. 510). Thompson (2016) asserts that understanding what a person means gives more insight into that person's thinking and actions than does understanding what they believe to be true. In general, Thompson and Harel's system addresses issues of understanding, meaning, and ways of thinking as presented in Table 1.

As characterized by Thompson and Harel (See Table 1), an understanding is a cognitive state of equilibrium, which may occur from assimilation to a scheme (i.e., stable understanding). According to Thompson et al. (2014), "A scheme, being stable, then constitutes the space of implications resulting from the person's assimilation of anything to it. The scheme is the meaning of the understanding that the person constructs in the moment" (p. 13). Alternatively, a cognitive state of equilibrium might be a state the "person has struggled to attain at that moment through functional accommodations to existing schemes . . . and is easily lost once the person's attention moves on" (i.e., in-the-moment understanding; Thompson et al., 2014, p. 13).

Table 1
Definitions of Understanding, Meaning, and Ways of Thinking (Thompson et al., 2014)

Construct	Definition
Understanding (in-the-moment)	Cognitive state resulting from an assimilation
Meaning (in-the-moment)	The space of implications existing at the moment of understanding
Understanding (stable)	Cognitive state resulting from an assimilation to a scheme
Meaning (stable)	The space of implications that results from having assimilated to a scheme. The scheme is the meaning.
Way of Thinking	Habitual anticipation of specific meanings or ways of thinking in reasoning

According to Thompson et al. (2014), "To construct a scheme, [an individual] must repetitively engage in the reasoning that will become solidified in that scheme in order to have occasions to develop the imagery that supports it" (p. 12). Furthermore, Thompson (1996) characterizes image as "being constituted by experiential fragments from kinesthesis, proprioception, smell, touch, taste, vision, or hearing" (p. 267), and may "entail fragments of past affective experiences, such as fearing, enjoying, or puzzling, and fragments of past cognitive experiences, such as judging, deciding, inferring, or imagining" (p. 268). Therefore, a person's images can be drawn from many sources and tend to be highly idiosyncratic (Thompson, 1994).

Whereas most prior studies utilizing Thompson and Harel's system (Harel, 2021; Moore & Thompson, 2015; Thompson, 2016) have focused on teachers' and students' mathematical meanings (e.g., angle measure, continuous variation, and function), recent research by Courtney (2021) examined grades 6-12 mathematics teachers' understandings of differentiated instruction and how attending to teachers' understandings can support interventions that impact these understandings in productive (in terms of student learning) ways. Courtney (2021) provides an example of one mathematics teacher's stable understanding of differentiated instruction as "to provide two types of problems (average and more challenging problems) and provide struggling students with extra support or extended time so that all students stay busy and complete work in the same amount of time" (p. 200)—an understanding that incorporates this teacher's beliefs, images, and judgments about her students, instructional design, and the dynamics of the classroom environment.

Although one might question how this stable understanding can be defined as a "cognitive" state resulting from an assimilation to a scheme (Thompson et al., 2014), it is important to note that Piaget had an expansive meaning of action, as "all movement, all thought, all emotion that responds to a need" (Piaget, 1968, p. 6). Therefore, as characterized by Thompson (2016), when "Piaget spoke of schemes, he had in mind organizations of mental and affective activity whose contents could be highly nuanced and could contain several layers of structure" (p. 432). For Piaget (1973), the "affective and cognitive mechanisms always remain interrelated though distinct" (p. 261). In a similar manner, for Vergnaud (2013), "[c]ognition is affective, or it is not, and affectivity is cognitive, or it is not. A scheme conveys both characteristics" (p. 52). Therefore, we interpreted the phrase "a cognitive state resulting from an assimilation to a scheme" as being inclusive of all cognitive, affective, and social activity. Such an interpretation aligns with Kihlstrom's (2016) depiction of a cognitive state as comprised of perceiving, remembering, knowing, and believing, and theory of mind research (Conway et al., 2019; Koster-Hale et al., 2017; Ruhl, 2020), which attribute mental states (i.e., beliefs, intents, desires, emotions, and kno wledge) to ourselves and others.

Methods

Since this study attempted to reveal teachers' understandings and explore how these conceptions impacted their teaching of developmental mathematics, it was necessary to make models of teachers' conceptions. The case study described in this paper expresses our second-order models (Steffe, 1995) of teachers' understandings at various points throughout the study; that is, throughout the activities (i.e., interventions) designed to make teachers' (schemes of) meanings explicit.

Participants

Participants were two tertiary mathematics teachers in the Midwestern United States. They were self-selected and met the following criteria: (a) university or community college mathematics teacher; (b) teaching at least one developmental mathematics course during summer 2021; and (c) prior experiences teaching developmental mathematics. Information regarding each participant's teaching experiences and course information are presented in Table 2.

Table 2
Participating Teacher Information

Participant/ Teaching Experience	Post-Secondary	Developmental Math,	Developmental Math Course(s) Taught, Summer 2021	Course Information	Student Information	Student Pass Rate, ≥ **70%** (Summer 2021)	Overall Student Pass Rate (Last 4 Years)
Instructor 1	5 years	4 years	Math Skills Workshop (2 sections)	6 weeks (Total 29 sessions) 5 days a week 75 minutes	30 Incoming Freshman (1 non-traditional)	25/39 (86.2%)	65/76 (85.5%)
Instructor 2	16 years	16 years	Beginning Algebra (1 section)	8 weeks (Total 13 sessions) 2 days a week 90 minutes	13 Non-traditional	11/13 (84.6%)	40/53 (75.5%)

Although participants' summer courses encompassed 6-8 weeks (see Table 2), the study itself was 12 weeks in length and included weekly group meetings before, during, and after the summer courses. In a national study using data from college transcripts, Attewell et al. (2006) found only 30% of all students pass all the developmental mathematics courses in which they enroll. This data, as well as the results from Chen (2016), indicate that the two participating teachers (or instructors) described here, with much higher student passing (or student success) rates, serve as models for effective developmental mathematics practices. Therefore, our attempts to characterize participating teachers' understandings has the potential to identify pedagogically powerful meanings related to developmental mathematics and its students.

Data Collection

The study employed a modified version of the *reflective investigation methodology* (Trouche et al., 2018) for data collection, which is naturally associated with case studies and grounded by five main principles: (a) broad collection of resources; (b) long-term follow up; (c) in- and out-of-class follow-up; (d) reflective follow-up; and (e) confronting a teacher's views on her documentation work. Due to the length of the summer courses and limits precluding in-person interactions, some principles could not be strictly adhered to, resulting in a modified version of the methodology. Specifically, each higher institution had policies prohibiting non-district personnel from observing in-person or remote instruction, thus precluding any form of classroom observation.

According to Trouche et al. (2018), the active involvement of the teacher is a practical necessity in reflective investigation methodology, as she "is the one having access to . . . her documentation work (beyond the direct observation of the researcher)" (p. 6). Furthermore, reflective investigation involves "teachers closely in the data collection . . . and accessing their 'thinking' through reflective analysis and stimulated recall" (Trouche et al., 2019). This notion is reinforced here in that the study's two participants (i.e., Instructors 1 and 2) are two of the authors.

Both instructors utilized student surveys at the beginning of their respective courses to obtain background information and information about their students' perceptions of mathematics. Instructor 1 included 5 Likert scale questions, with a range of 1 (strongly agree) to 5 (strongly disagree). Instructor 1's survey asked students about their views of and abilities with mathematics and included questions such as: "I like math," "Math makes me stressful," and "I think I'm good at math." Students were also asked to identify specific pre-Algebra and Algebra topics they believed they would struggle with or did not recall (e.g., order of operations, solving systems of linear equations). Instructor 2's survey asked students to complete open-ended questions through email or during a virtual meeting. The survey was posted as a Blackboard assignment and asked students a variety of questions, including: "How do you feel about taking a math course synchronously/online?"; "What are your strengths and weaknesses in mathematics?"; and questions regarding students' experiences with the institution's videoconferencing application (i.e., Webex) and learning management system (i.e., Blackboard).

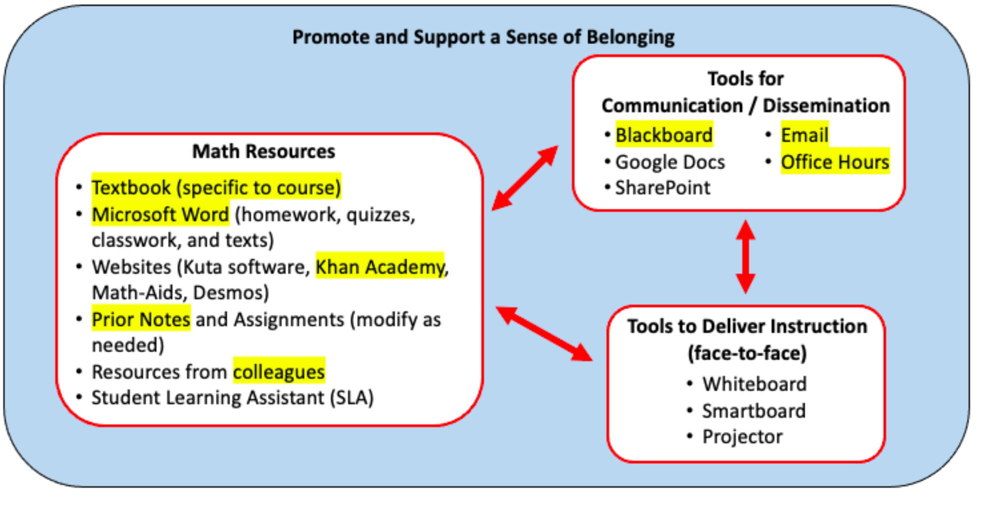
The study's data corpus consisted of participants' semi-weekly self-recorded videos discussing their lesson plans—with accompanying documents and internet links to all materials the teachers utilized during instruction and assessment—and their reflections of lesson implementations. In addition, weekly online discussions with both participants were designed to probe teachers' understandings in more detail. These sessions were reflective in nature to discuss the weekly successes and potential modifications needed to increase student learning and engagement. Therefore, data also comprised video recordings and field notes from these weekly discussions. Finally, data included two tools specific to reflective investigation methodology: reflective mappings of each teacher's resource system (RMRS) and inferred mappings of each teacher's resource system (IMRS). According to Wang (2018), an RMRS is a methodological tool created by a teacher where the teacher is asked to "draw a map to present her resources in a structured way based on her own reflection" (p. 197). Similarly, an IMRS is a methodological tool created by "the researcher based on the observations of and interviews with the teachers about their resource work" (Rezat et al., 2019, pp. 357-358).

Data Analysis

Making sense of tertiary teachers' understandings of developmental mathematics and its students required us to develop models of teachers' ways of operating—models that represented our interpretations of teachers' interactions with study activities and digital resources. Using data generated from study activities and reflective investigation, these models were tested, modified, and refined through ongoing and retrospective conceptual analyses (Thompson, 2008; von Glasersfeld, 1995) of the data corpus.

As part of the group meetings during Week 5 (Instructor 1) and Week 6 (Instructor 2), participants described their initial RMRS. During these meetings we questioned each instructor to clarify any portions of their RMRS that were unclear. During the Week 7 meeting, the first author created an inferred mapping of each teacher's resource system (IMRS), based on each participant's RMRS and the discussions that occurred during these meetings. During the Week 9 meeting, each participant was confronted with the IMRS created by the first author and their original RMRS. We use the term "confront" here in the sense of Brousseau (1997), and as employed in the documentational approach, to mean "a focused comparison, bringing together for careful comparison" (Interglot, 2021). This meeting resulted in the two refined IMRS shown in Figures 1 and 2. These figures illustrate participants' resources partitioned into three groups: resources specific to mathematics content, tools for communication and dissemination, and tools to deliver instruction. Neither participant's initial RMRS included the arrows

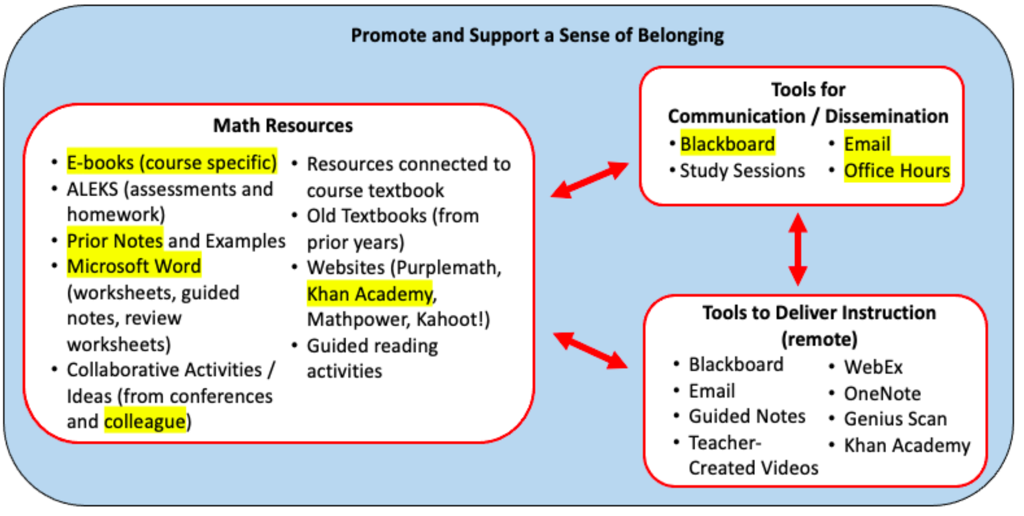
Figure 1
Refined Inferred Mapping of Instructor 1's Resource System (IMRS)



Participants' comments during group meetings emphasized their integration of these three types of resources. For example, whenever participants' spoke about the "Math Resources" they used, they invariably included ideas about and a rationale for utilizing "Tools for Communication and Dissemination" and "Tools to Deliver Instruction." Resources used by both participants are highlighted in Figures 1 and 2, and indicate that regardless of course format (i.e., face-to-face, remote), certain communication and dissemination tools and mathematics content resources were useful to each participant.

Figure 2

Refined Inferred Mapping of Instructor 2's Resource System (IMRS)



A discussion during the Week 11 meeting indicated Blackboard was used by both participants, because it was the learning management system for each institution. Furthermore, each participant's respective institution mandated their use of email and office hours. Finally, each teacher's use of the course textbook, whether e-book or physical copy, is not uncommon as textbooks "appear to be a predominant feature in the teaching of tertiary mathematics" (Mesa & Griffiths, 2012, p. 85). Although participants' common use of notes, Khan Academy, Microsoft Word, and their colleagues will be addressed further in the Findings and Discussion section, each participant indicated their selection and use of specific resources were "dependent on their students' needs . . . [and] were provided to help develop, promote, and support a caring community in the classroom" (Instructor 1, Week 11 meeting).

Throughout the data corpus, participants indicated "their students' needs" referred to each student's individual circumstances and challenges, which included (Goldin et al., 2016; Hannula et al., 2019; Philipp, 2007): mathematics anxiety – "a few of these students are so afraid of math" (Instructor 1, Week 2 meeting) and "my students are so intimidated and scared of mathematics because of bad prior experiences" (Instructor 2, Week 2 meeting); test anxiety – "testing is always a traumatic experience for these students" (Instructor 2, Week 9 meeting); coronavirus anxiety – "many of my students don't necessarily feel safe meeting in person" (Instructor 1, Week 2 meeting); student emotions – "my second class [of students] . . . really hate fractions more than my first class [of students] . . . so, I always make sure to include more fractions because they need that practice" (Instructor 1, Week 11 meeting); low mathematics self-efficacy – "several students just believe they will never understand math" (Instructor 1, Week 11 meeting) and "part of my job is to get these students to believe they can actually do the math" (Instructor 2, Week 11 meeting); poor motivation – "too many students will not even attempt challenging problems (Instructor 1, Week 5 meeting); and frustration – "many of my students with families and work get fed up trying to navigate school" (Instructor 2, Week 6 meeting).

Hackenberg (2010) defines a *mathematical caring relation* as a "quality of interaction between a student and a mathematics teacher that conjoins affective and cognitive realms in the process of aiming for mathematical learning" (pp. 57-58). Therefore, participating teachers enacted their intent to "support a caring community in the classroom" (i.e., "support mathematical caring relations in the classroom") through being attentive to the needs of their students; that is, through listening to and observing their students; demonstrating respect for their students; thinking and reflecting on ways to support their students; being approachable, available, and responsive to their students; and creating an environment where caring relations can flourish (Benken et al., 2015; Miller et al., 2017; Noddings, 2012).

Participants' comments during group meetings also emphasized an image that, along with being integrated, their resources were always in the foreground of their practice, while promoting and supporting a sense of belonging in the mathematics classroom was always in the background. Whether thinking about and searching for activities or problems for an assessment, disseminating information via email, or using the class whiteboard for students to solve problems, each action attempted to promote and support a sense of belonging in the mathematics classroom for their students. Specific participant comments and assertions that illustrate an image of promoting and supporting their students' sense of belonging in the classroom, included (Boaler & Selling, 2017; Dweck, 2006; Goldin et al., 2016; Hannula et al., 2019; Philipp, 2007; Rott et al., 2018): mathematical identity – "I want to make sure each student believes they can be . . . they are a contributor to the mathematics classroom . . . to view themselves as mathematicians" (Instructor 2, Week 6 meeting); motivation and persistence – "I usually focus on motivation and persistence as a way to promote students' beliefs in their ability to contribute to class discussions" (Instructor 1, Week 5 meeting); interest – "pair and small group activities are useful to support student interest and promote a sense of togetherness" (Instructor 1, Week 3 meeting); growth mindset – "students need to believe they can not only be successful in my class, but have the abilities to be successful in their future math courses" (Instructor 1, Week 5 meeting); and values – "since most of my classes have both traditional and nontraditional students, I have to make certain to provide activities that encourage each group to value mathematics" (Instructor 2, Week 9 meeting).

According to Thompson and Harel (Thompson et al., 2014) (See Table 1), the meaning of an understanding is the space of implications the current understanding mobilizes—the actions or schemes the current understanding implies (i.e., brings to mind with little effort). Therefore, as illustrated above, the meaning of participating teachers' understandings "to develop, promote, and support a caring community in the classroom, dependent on my students' needs" and "to promote and support in my students, a sense of belonging as learners of and contributors to mathematics"—what comes to participating teachers' minds in situations related to developmental mathematics and its students—are affective aspects of their students (i.e., anxiety, beliefs, emotions, feelings, frustration, interest, identity, mindset, motivation, persistence, self-efficacy, values). Goldin (2002) defines beliefs as "internal representations to which the holder attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured" (p. 61). For Philipp (2007) beliefs are "the lenses through which one looks when interpreting the world" (p. 258). Therefore, participating teachers' stable understandings related to developmental mathematics and its students are indicative of their beliefs about developmental mathematics and its students.

Findings and Discussion

In this section, we address the three research questions, paying particular attention to the notions of mathematical caring relations and sense of belonging as learners of and contributors to mathematics identified in our analysis.

Teachers' Understanding of Developmental Mathematics

Both instructors exhibited meanings for developmental mathematics that involved more than just a series of middle grade and high school mathematics topics (e.g., operations of real numbers, order of operations, and applications of linear equations); rather, as described by Instructor 1, "developmental math involves content that is foundational to my students' future mathematics learning" (Week 10 meeting). Instructor 1 further indicated that since she had taught or tutored each class for which developmental mathematics is a prerequisite, she needed to "prepare all of [her students] for each of these different classes." For example, since Statistics was the next course some of her students would take, she found space in the syllabus to introduce basic concepts in statistics—even though this topic had been phased out of the developmental mathematics curriculum. Instructor 1 asserted, "I want my students to succeed in all math classes, not just this one" (Week 3 meeting). Instructor 2 also indicated a desire to prepare students for subsequent mathematics courses by helping her students "build or strengthen their mathematics foundation for their next course, including study strategies" (Week 3 meeting). Finally, as indicated in the Data Analysis section, participating teachers focused on their students' circumstances and challenges.

Therefore, as characterized by Thompson and Harel (See Table 1), participating teachers' stable understanding of developmental mathematics—that is, participating teachers' cognitive state of equilibrium occurring from assimilation to a scheme (of meanings)—is "*content foundational to students' subsequent math courses and dependent on each student's needs*." Since meaning is implicative (Piaget & Garcia, 1991), participating teachers' meaning of developmental mathematics has implications for their further action. Thus, the participating teachers' actions focus on highlighting the "foundational to subsequent mathematics courses" aspect of development mathematics, rather than "a remedy for missing, weak, or fragile knowledge" conception, and are "dependent on each student's needs" (i.e., anxiety, beliefs, emotions, feelings, frustration, interest, identity, motivation, persistence, self-efficacy, values).

Teachers' Understanding of the Students Enrolled in Developmental Mathematics

As described earlier, each teacher provided their students with an initial survey in their respective courses and asked their students how they were thinking about the course (e.g., what they were struggling with, what was useful) throughout the summer session. Each teacher developed models of their students based on their interpretations of students' responses to these queries. For example, Instructor 2 had a student who had not taken a mathematics class for over seven years assert she had "a lot of college credits, but [had] just given up on myself . . . [and] wanted to prove to myself and my children that it's never too late." As such, Instructor 2 made a point to "check-in" with this student on a regular basis to make certain she was feeling comfortable and achieving her goals. The survey for Instructor 1 indicated more than half of the students reported mathematics as stressful and a subject they disliked. Instructor 1 indicated "understanding that her students feel stressed impacts how [her] class is operated" (Week 9 meeting). Instructor 1 further asserted "creating a welcoming environment and providing extra resources can help students see the subject matter through a different lens" (Week 9 meeting). Finally, Instructor 1 indicated a goal for her course is for "students to leave the course no longer 'hating' math. They do not need to love it, but they don't hate it" (Week 9 meeting).

Since Instructor 2 taught remotely, she provided a live tutorial with her class where she went through the Blackboard and Webex environments, making certain students had a sense for where to find the calendar, weekly and daily folders, assignments, assessments, video links, and chat box. Instructor 2 also provided her students with an unlimited amount of time to complete online quizzes, since she wanted to "make certain students feel like they have enough time to do really good work, to be thorough, and to show all of their work." (Week 10 meeting). Instructor 2 indicated this action helped to deter some mathematics anxiety, because students are not watching the clock. According to Instructor 2, "If I can reduce a little bit of their anxiety by taking away the time constraint, then I will" (Week 10 meeting). Instructor 2 also asked her students to show their work on paper—since her ALEKS assessments did not have such capacity—and send her their work as a pdf or image via email. Instructor 2 used student work to look for careless errors and provide students with partial credit. Finally, Instructor 2 promoted a collaborative and supportive environment by encouraging students to use "clapping" and "thumbs up" emojis to show appreciation for student or group virtual demonstration work.

Both teachers utilized their understanding of their students' future courses to emphasize concepts and behaviors important to students' continued success. In addition, participating teachers' stable understandings related to situations involving developmental mathematics and its students comprised "developing, promoting, and supporting mathematical caring relations in the classroom, dependent on their students' needs" and "promoting and supporting a sense of belonging as learners of and contributors to mathematics." Therefore, participating teachers understand their students (i.e., students enrolled in developmental mathematics) as "*individuals with varying mathematics and school experiences in need of a caring environment and a sense of belonging as learners of and contributors to mathematics*"—indicative of participating teachers' beliefs about their students. Finally, since meanings are implicative (Piaget & Garcia, 1991), participating teachers' actions focused on affective aspects of their students (e.g., anxiety, beliefs, emotions, feelings, frustration, interest, identity, motivation, persistence, self-efficacy, values).

Teachers' Understanding Resources Utilized to Support Developmental Math Instruction

Although the specific resources each teacher utilized was dependent on whether classes were face-to-face or remote, both teachers selected resources that would provide their students with opportunities to engage in mathematics in a caring environment that supported students' sense of belonging as learners of and contributors to mathematics. This is an example of the process of instrumentalization, where participating teacher's understandings guided the choices they made among various resources and the way these resources were appropriated.

This way of thinking is demonstrated by examining those resources both participants utilized to find and generate mathematics content in their respective courses and environments (i.e., Khan Academy, Microsoft Word, their colleagues, and their prior notes; see Figures 1 and 2). Regarding Khan Academy, Instructor 1 indicated the resource offered her students "free practice over many topics." Furthermore, although she had not integrated the resource into her summer course as much as she had when classes were fully remote (over the prior year due to the pandemic), she continued to provide her students with links to Khan Academy Pre-Algebra and Algebra topics that aligned with her curriculum. Instructor 2 indicated another reason why she utilized Khan Academy, which was because most of her students were familiar with it, either through a prior high school mathematics course or their own children's use of the resource.

Regarding use of their prior notes, Instructor 2 indicated her students had found her notes, especially her guided notes, to be quite beneficial during asynchronous classes over the past year. As such, Instructor 2 decided to utilize these same notes in her current synchronous setting. Finally, Instructor 2 provided self-made video tutorials corresponding to her guided notes, again prepared over the past year, as an additional resource to support her students. Instructor 1 indicated she provided her students with pre-made notes with the word problems from each section written out and a corresponding blank space for student work below each problem. According to Instructor 1, "this allows students to focus on engaging with the problem, the mathematics, rather than spending time writing out a bunch of words." Instructor 1 believed this support was especially important since her students did not have their own copies of the textbook. Finally, each teacher utilized Microsoft Word to create their assignments, quizzes, tests, and notes. Both teachers agreed they utilized Word because it made their documents easier to read than handwritten work, easily incorporated mathematics equations, formulas, and symbols through Equation Editor, and made such documents "simply look nicer." Instructor 2 added that Word documents could easily be changed to pdf format and uploaded to Blackboard Ally, a tool that generates alternative formats for students to download, provides accessibility scores, and gives instructor feedback on how to improve their accessibility score. Instructor 2 indicated that making digital course content more accessible to all her students supported her attempts to support student engagement and sense of belonging. Therefore, the participating teachers understood these resources as "*tools to support each student, based on their individual needs, as they engage with mathematics and interact with their classmates and themselves (i.e., the teacher)*"—indicative of participating teachers' beliefs about developmental mathematics and its students. Finally, since meanings are implicative (Piaget & Garcia, 1991)—and as a result of the instrumentalization process—the participating teachers' actions with respect to these tools focused on their students' "individual needs" (e.g., anxiety, beliefs, emotions, feelings, frustration, interest, identity, motivation, persistence, self-efficacy, values).

Conclusion

This study investigated tertiary mathematics teachers' understandings of developmental mathematics and its students and the resources teachers utilize to support instruction. As characterized in our analysis, participating teachers' stable understandings (i.e., teachers' cognitive state resulting from assimilation to a scheme) involved "promoting and supporting mathematical caring relations and a sense of belonging as learners of and contributors to mathematics." Furthermore, the participating teachers' schemes (i.e., meanings), which constitute the space of implications resulting from teachers' assimilation to these schemes of meanings, were indicative of teachers' beliefs and focused on affective aspects of their students' varied needs and experiences. Aligned with Hackenberg (2005), we posit that participating teachers' meanings involved a scheme of decentering (Piaget, 1962), where decentering is the attempt to imagine one's experience from another perspective (Steffe & Thompson, 2000). A teacher's capacity to decenter not only impacts their attention to building and supporting mathematical caring relations (Hackenberg, 2005), but also "such things as teachers' decisions to pose (or not pose) a question, the nature of teachers' questions, the quality of their explanations, and their choices for student contributions" (Teuscher et al., 2016, p. 453).

Although it seems self-evident that mathematics teachers care for their students and attempt to provide students with a sense of belonging to the classroom, some students' fail to perceive their teachers as promoting mathematical caring relations or a sense of belonging as a learner of and contributor to mathematics. Meyers (2009) asserts that some "faculty members doubt that caring has a place in college-level instruction . . . that it is more appropriate for younger children . . . [and] that caring implies the absence of academic rigor or lowered expectations for students" (pp. 207-208). In addition, a recent study by Gallup and Purdue University (Gallup-Purdue Index, 2015), designed to explore the relationship between the college experience and college graduates' jobs and lives, found that only 27% of college graduates (n = 31,117) strongly agreed that "they had a professor who cared about them personally" (p. 10). Finally, findings from a study by Hodara (2019) of 24,766 students indicated that "developmental mathematics students reported slightly lower sense of belonging, confidence, and mental health ratings compared to their peers who did not take developmental education in the first year of college" (p. 14).

Given this case study only included two participants, our findings are not generalizable. The study's small sample size and lack of range within teaching styles are a few additional notable limitations. Therefore, future research should explore the understandings of a larger sample of tertiary teachers of developmental mathematics to determine the meanings by which teachers operate; specifically, the ubiquity of schemes of decentering. Such research should also include examination of the types of professional learning experiences most propitious to fostering schemes of decentering in tertiary mathematics teachers.

As demonstrated here, networking the documentational approach to didactics and a theory of meanings allowed us to develop viable models of tertiary teachers' understandings of developmental mathematics and its students and the resources they utilize to support instruction. These frameworks were initially chosen to support our attempts to make sense of an empirical phenomena (i.e., teachers' understandings), but the results provide researchers and mathematics teacher educators with a framework focused on teachers' documentation work that supports identification of the schemes of meanings with which teachers teach—including teachers' belief structures. Furthermore, the framework has the potential to also support the design of productive professional learning experiences for both preservice and inservice teachers. Finally, such research should explore the role the instrumentation process might play in transforming teachers schemes of meanings, where the features and teachers' understanding of a resource or set of resources impact that teacher's schemes as a result of their interactions with the resource(s).

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